

4.1

Triangles and Angles

40

- Goals**
- Classify triangles by their sides and angles.
 - Find angle measures in triangles.

VOCABULARY

Triangle

Vertex

Adjacent sides

Legs

Hypotenuse

Base

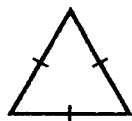
Interior angles

Exterior angles

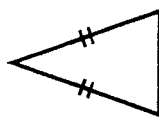
Corollary

NAMES OF TRIANGLES

Classification by Sides



3 congruent sides



At least 2 congruent sides

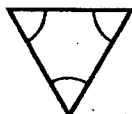


No congruent sides

Classification by Angles



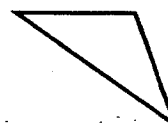
3 acute angles



3 congruent angles



1 right angle

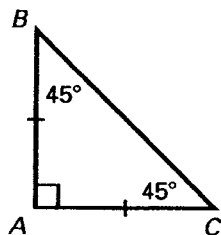


1 obtuse angle

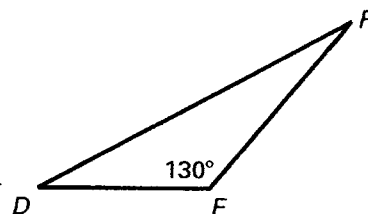
Example 1 Classifying Triangles

Classify each triangle. Be as specific as possible.

- a. ABC has two acute angles, one right angle and two congruent sides. It is a _____.



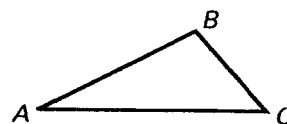
- b. DEF has one obtuse angle and no congruent sides. It is an _____.

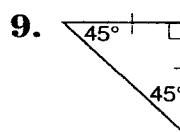
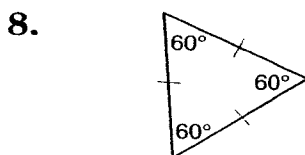
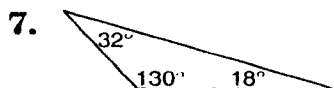
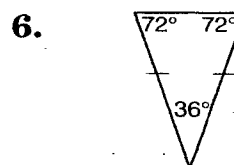
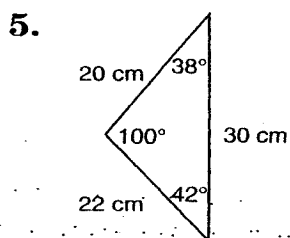
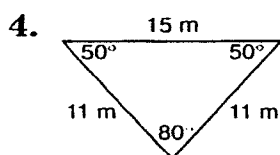
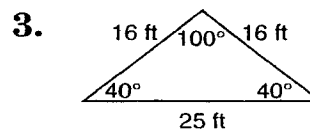
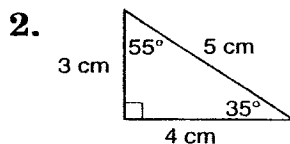
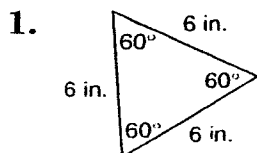


THEOREM 4.1: TRIANGLE SUM THEOREM

The sum of the measures of the interior angles of a triangle is _____.

$$m \angle A + m \angle B + m \angle C = \underline{\hspace{2cm}}$$



Skills Practice *4.1 Blue***Classifying Triangles****Classify each triangle by its angles and by its sides.****Make a sketch of each triangle. If it is not possible to sketch the figure, write not possible.**

10. right scalene

11. obtuse isosceles

12. right isosceles

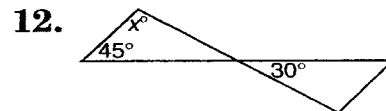
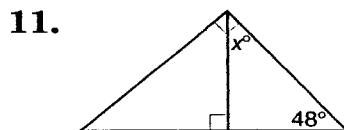
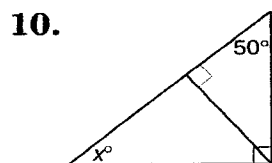
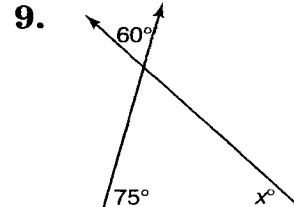
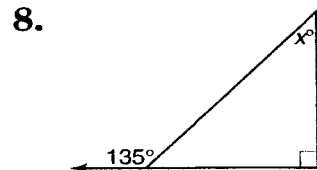
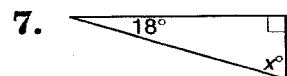
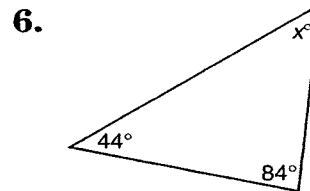
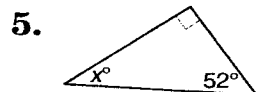
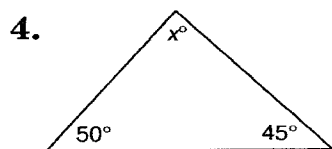
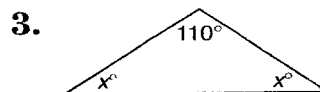
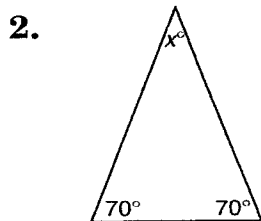
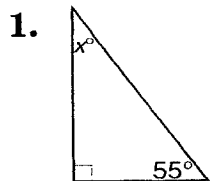
13. right equilateral

Skills Practice

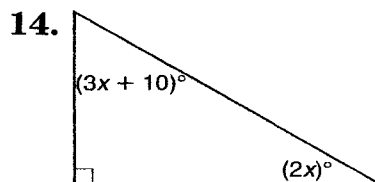
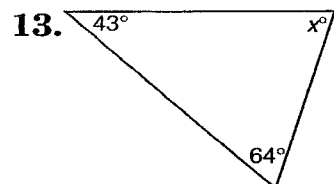
4.161ax

Angles of a Triangle

Find the value of each variable.



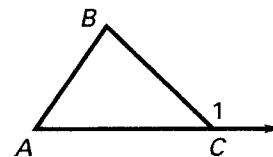
Find the measure of each angle in each triangle.



THEOREM 4.2: EXTERIOR ANGLE THEOREM

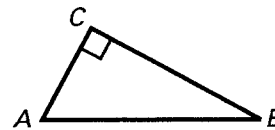
The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

$$m \angle 1 = \underline{\hspace{1cm}} + \underline{\hspace{1cm}}$$

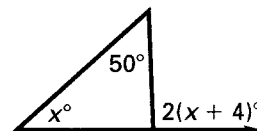
**COROLLARY TO THE TRIANGLE SUM THEOREM**

The acute angles of a right triangle are complementary.

$$m \angle A + m \angle B = \underline{\hspace{1cm}}$$

**Example 2** *Finding an Angle Measure*

You can apply the Exterior Angle Theorem to find the measure of the exterior angle shown. First write and solve an equation to find the value of x :



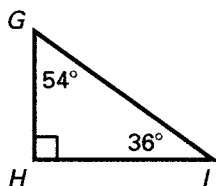
$$x^\circ + 50^\circ = 2(x + 4)^\circ \quad \text{Apply the Exterior Angle Theorem.}$$

$$\underline{\hspace{1cm}} = x \quad \text{Solve for } x.$$

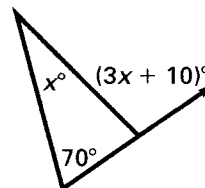
Answer So, the measure of the exterior angle is $2 \cdot (\underline{\hspace{1cm}} + 4)^\circ$, or $\underline{\hspace{1cm}}^\circ$.

✓ Checkpoint Complete the following exercises.

- 1.** Classify the triangle by its angles and by its sides.



- 2.** Find the measure of the exterior angle shown.



Practice

4, 1 Blue

Exterior Angle Theorem

Name the angles.

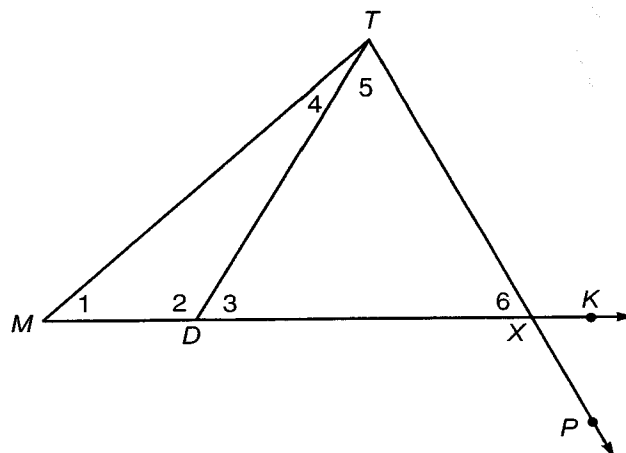
1. an interior angle of $\triangle MDT$

2. an interior angle of $\triangle TDX$

3. an exterior angle of $\triangle MTX$

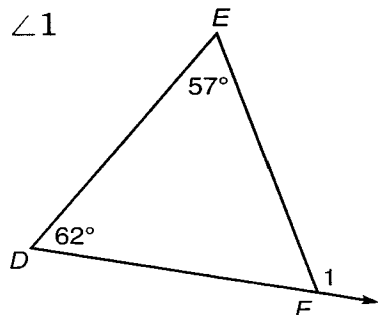
4. an exterior angle of $\triangle TDX$

5. a remote interior angle of $\triangle TDX$ with respect to $\angle 2$

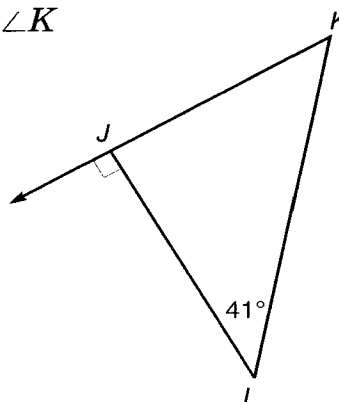


Find the measure of each angle.

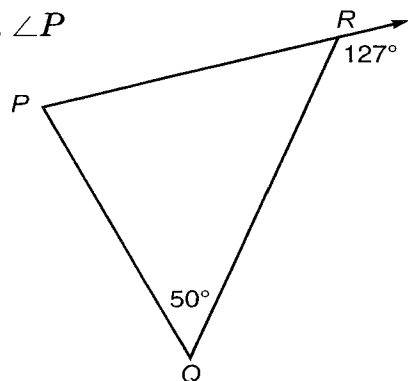
6. $\angle 1$



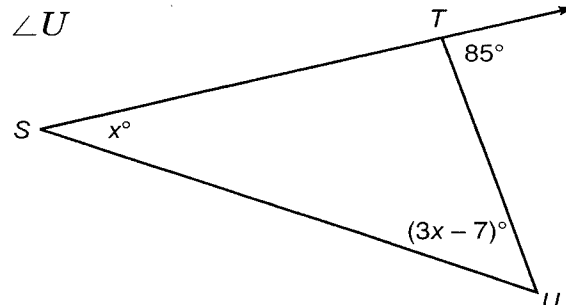
7. $\angle K$



8. $\angle P$



9. $\angle U$



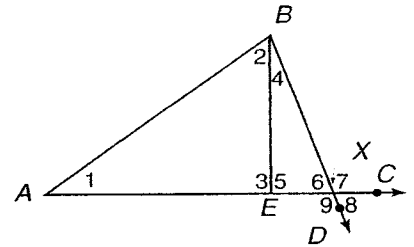
Skills Practice

461 Blue

Exterior Angle Theorem

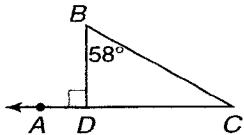
Name the angles.

1. an exterior angle of $\triangle ABX$
2. an exterior angle of $\triangle BEA$
3. an interior angle of $\triangle ABX$
4. an interior angle of $\triangle BEA$
5. a remote interior angle of $\triangle ABX$ with respect to $\angle 7$
6. a remote interior angle of $\triangle BEA$ with respect to $\angle 5$

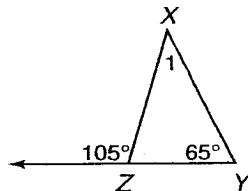


Find the measure of each angle.

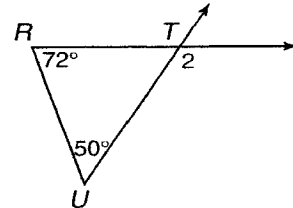
7. $\angle C$



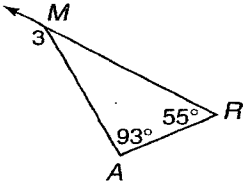
8. $\angle 1$



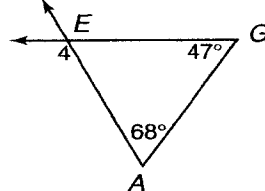
9. $\angle 2$



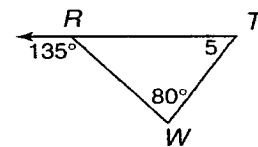
10. $\angle 3$



11. $\angle 4$



12. $\angle 5$



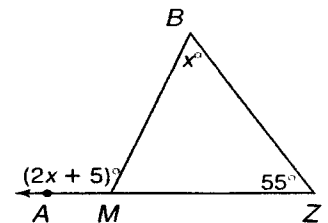
Use the figure at the right.

13. Find the value of x .

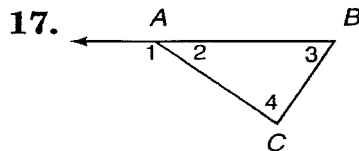
14. Find $m\angle B$.

15. Find $m\angle BMA$.

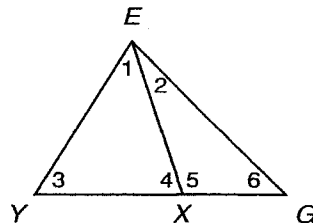
16. Find $m\angle BMZ$.



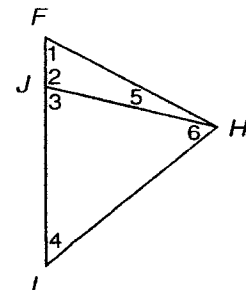
Replace each \bigcirc with $<$, $>$, or $=$ to make a true sentence.



18.



19.



$$m\angle 3 \bigcirc m\angle 1$$

$$m\angle 5 \bigcirc m\angle 3$$

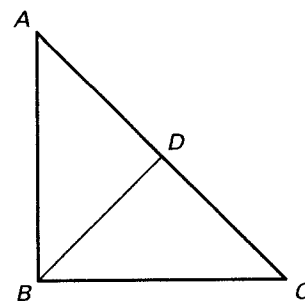
$$m\angle 2 \bigcirc m\angle 4 + m\angle 6$$

Practice A

For use with pages 194–201

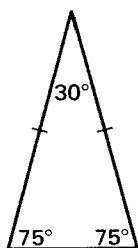
In the figure, $\overline{AB} \cong \overline{BC}$ and $\overline{BD} \perp \overline{AC}$. Complete the sentence.

- \overline{BC} is the _____ of right triangle $\triangle BDC$.
- \overline{BD} is a _____ of right triangle $\triangle BDC$.
- \overline{AC} is the _____ of isosceles triangle $\triangle ABC$.
- The legs of isosceles triangle $\triangle ABC$ are _____ and _____.
- The legs of right triangle $\triangle ADB$ are _____ and _____.

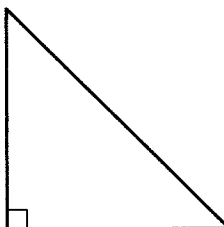


Classify the triangle by its angles and by its sides.

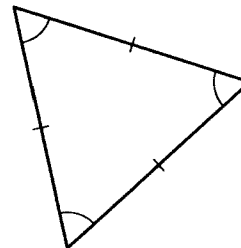
6.



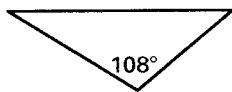
7.



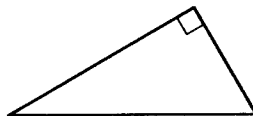
8.



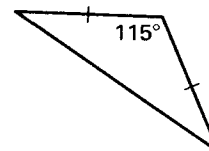
9.



10.



11.

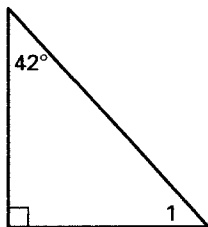


Classify the sentence with *always*, *sometimes*, or *never*.

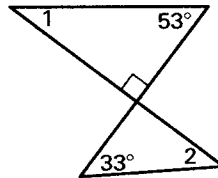
- An equilateral triangle is _____ an isosceles triangle.
- An isosceles triangle is _____ an equilateral triangle.
- A right triangle is _____ an acute triangle.
- An exterior angle of a triangle is _____ acute.

Find the measure of the numbered angle.

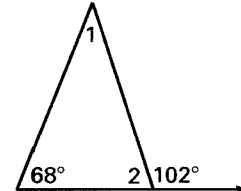
16.



17.



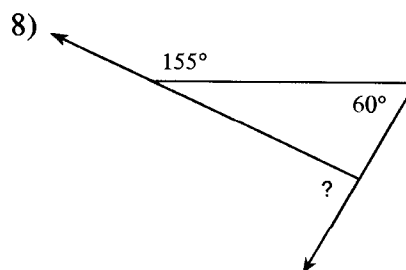
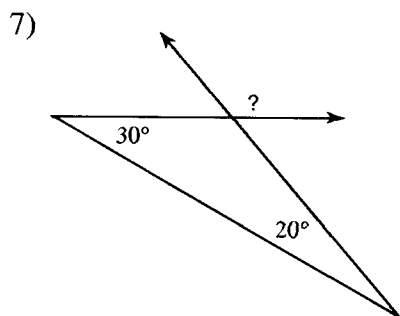
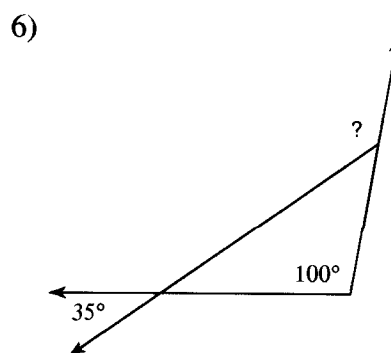
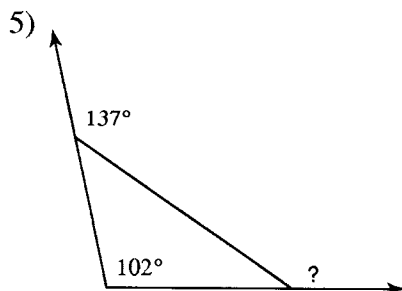
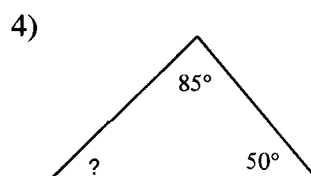
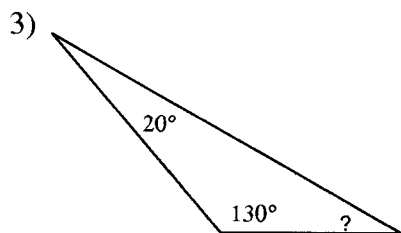
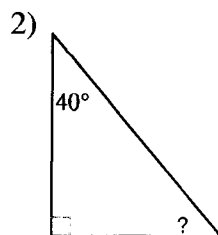
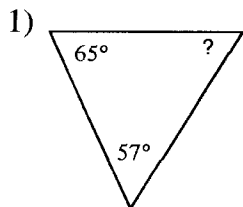
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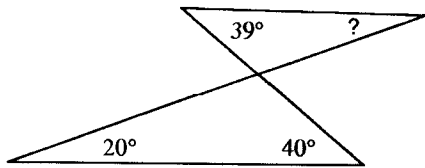
Angles in a Triangle

Date _____ Period _____

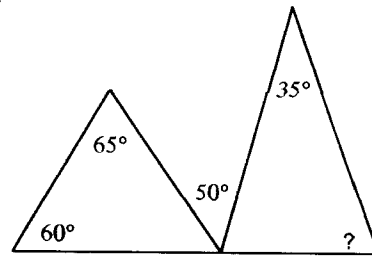
Find the measure of each angle indicated.



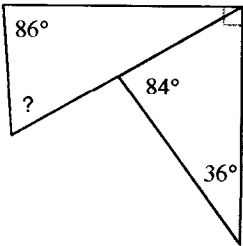
9)



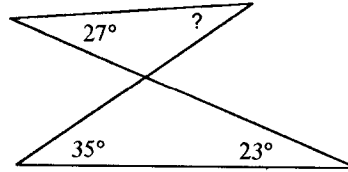
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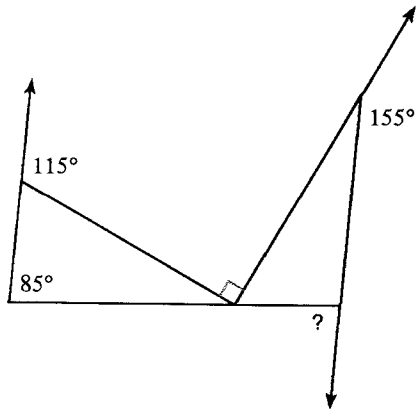
11)



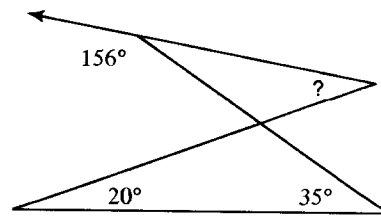
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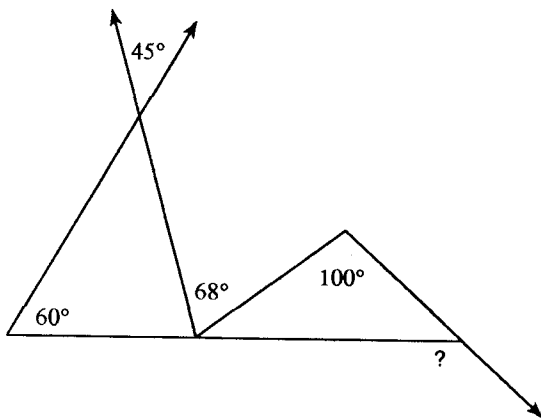
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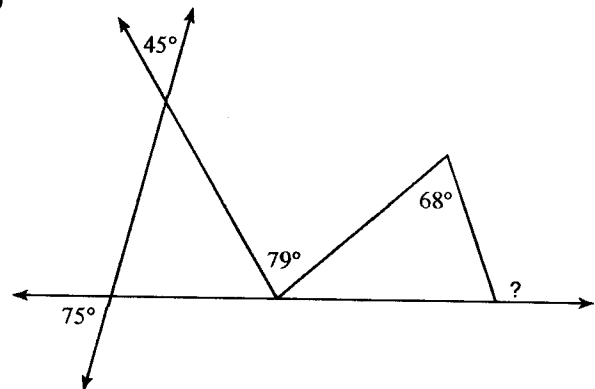
14)



15)

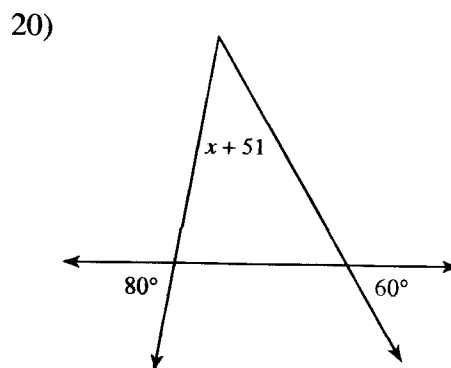
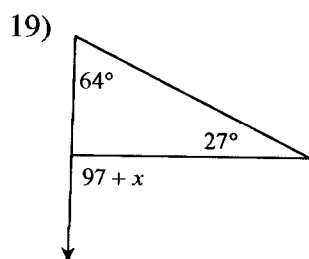
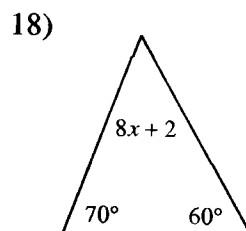
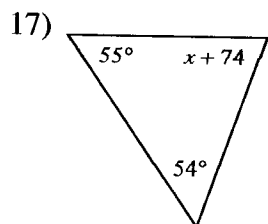


16)

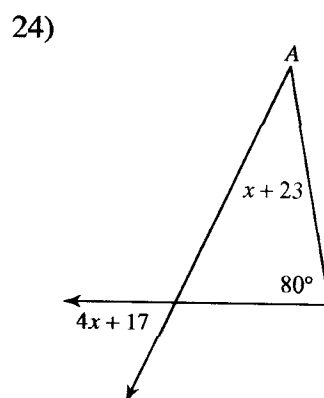
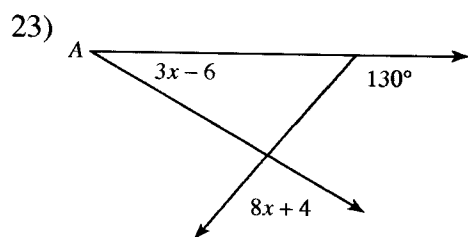
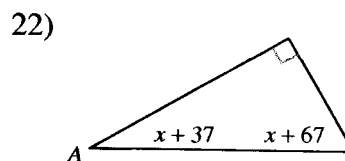
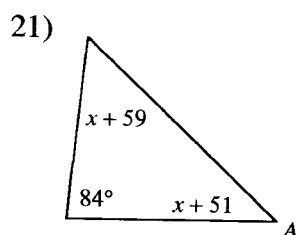


Solve for x .

64B



Find the measure of angle A.



4.2

Congruence and Triangles

46

- Goals**
- Identify congruent figures and corresponding parts.
 - Prove that two triangles are congruent.

VOCABULARY

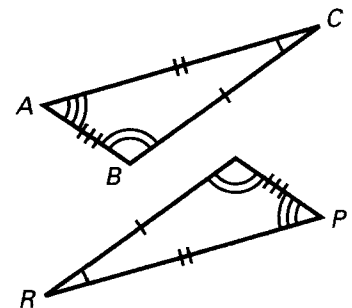
Congruent

Corresponding angles

Corresponding sides

Example 1 Naming Congruent Parts

Write a congruence statement for the triangles. Identify all pairs of congruent corresponding parts.



Solution

The diagram indicates that $\triangle ABC \cong \triangle RQP$.

The congruent angles and sides are as follows.

Angles: $\angle A \cong \angle R$, $\angle B \cong \angle Q$, $\angle C \cong \angle P$

Sides: $AB \cong RQ$, $BC \cong QP$, $AC \cong RP$

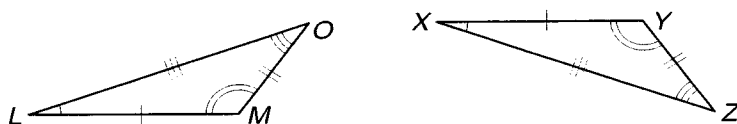
Study Guide

4.2 Blue

Congruent Triangles

When two figures have exactly the same shape and size, they are said to be congruent. For two congruent triangles there are three pairs of corresponding (matching) sides and three pairs of corresponding angles. To write a correspondence statement about congruent triangles, you should name corresponding angles in the same order. Remember that congruent parts are marked by identical markings.

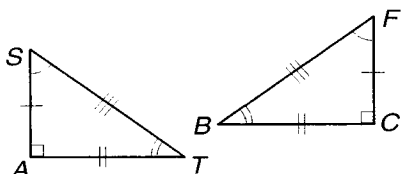
Example: Write a correspondence statement for the triangles in the diagram.



$$\triangle LMO \cong \triangle XYZ$$

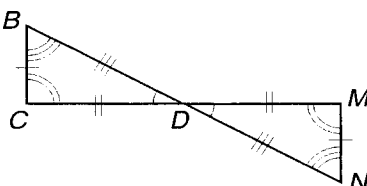
Complete each correspondence statement.

1.



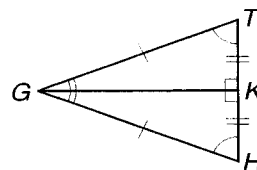
$$\triangle SAT \cong \triangle \underline{\hspace{1cm}}$$

2.



$$\triangle BCD \cong \triangle \underline{\hspace{1cm}}$$

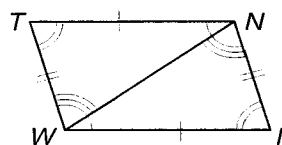
3.



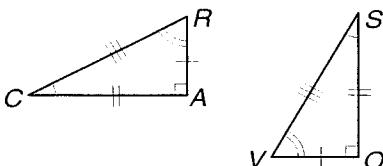
$$\triangle GHK \cong \triangle \underline{\hspace{1cm}}$$

Write a congruence statement for each pair of congruent triangles.

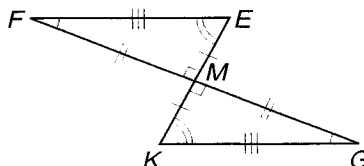
4.



5.



6.



Draw triangles $\triangle EDG$ and $\triangle QRS$. Label the corresponding parts if $\triangle EDG \cong \triangle QRS$. Then complete each statement.

7. $\angle E \cong \underline{\hspace{1cm}}$

8. $\overline{DG} \cong \underline{\hspace{1cm}}$

9. $\angle EDG \cong \underline{\hspace{1cm}}$

10. $\overline{GE} \cong \underline{\hspace{1cm}}$

11. $\overline{ED} \cong \underline{\hspace{1cm}}$

12. $\angle EGD \cong \underline{\hspace{1cm}}$

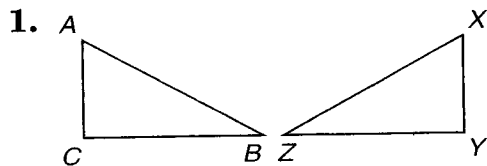
Skills Practice

4.2 Blue

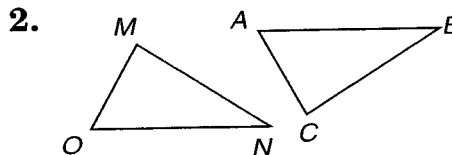
67

Congruent Triangles

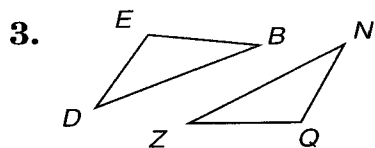
Name the congruent angles and sides for each pair of congruent triangles. Then draw arcs and slash marks to show the congruent angles and sides.



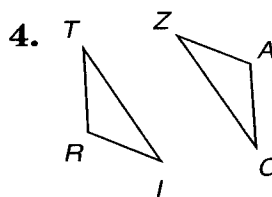
$$\triangle ACE \cong \triangle XYZ$$



$$\triangle MNO \cong \triangle CBA$$

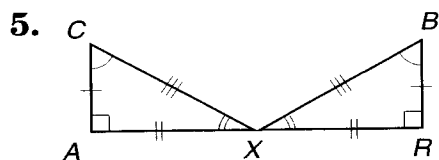


$$\triangle BDE \cong \triangle ZNQ$$

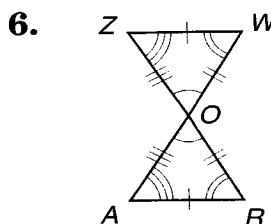


$$\triangle TRI \cong \triangle ZAC$$

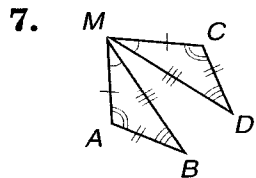
Complete each congruence statement.



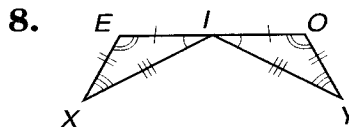
$$\triangle CAX \cong \triangle \underline{\hspace{1cm}} ?$$



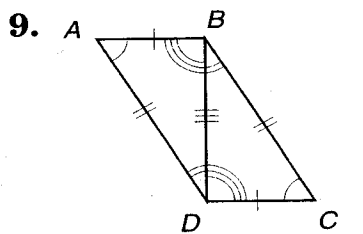
$$\triangle ZWO \cong \triangle \underline{\hspace{1cm}} ?$$



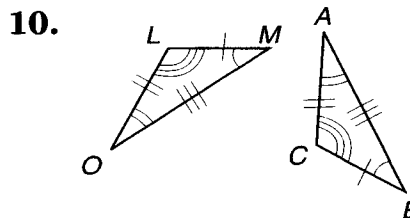
$$\triangle MAB \cong \triangle \underline{\hspace{1cm}} ?$$



$$\triangle EIX \cong \triangle \underline{\hspace{1cm}} ?$$



$$\triangle ABD \cong \triangle \underline{\hspace{1cm}} ?$$



$$\triangle LMO \cong \triangle \underline{\hspace{1cm}} ?$$

Practice

4.2 Blue

Congruent Triangles

If $\triangle RST \cong \triangle ABC$, use arcs and slash marks to show the congruent angles and sides. Complete each congruence statement.

1. $\angle C \cong$?

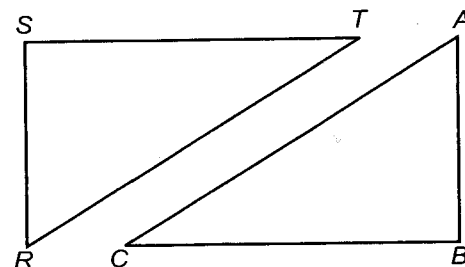
2. $\angle R \cong$?

3. $\overline{AC} \cong$?

4. $\overline{ST} \cong$?

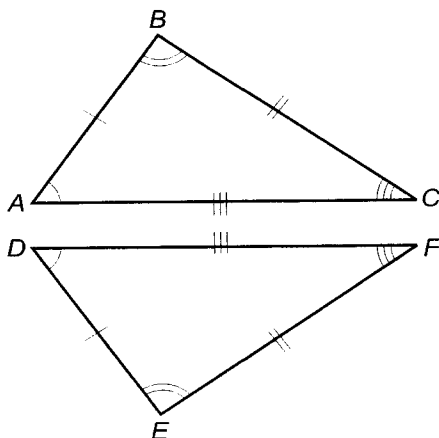
5. $\overline{RS} \cong$?

6. $\angle B \cong$?



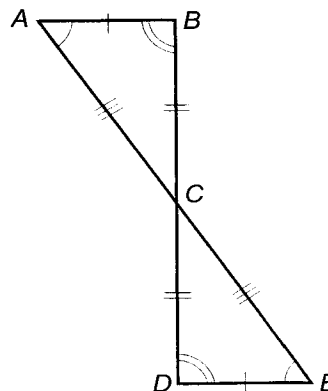
Complete each congruence statement.

7.



$\triangle ABC \cong \triangle$?

8.



$\triangle ACB \cong \triangle$?

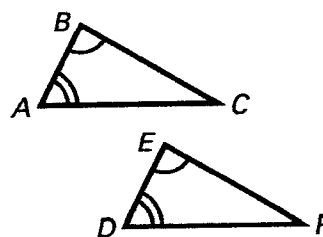
9. Given $\triangle ABC \cong \triangle DEF$, $AB = 15$, $BC = 20$, $AC = 25$, and $FE = 3x - 7$, find x .

10. Given $\triangle ABC \cong \triangle DEF$, $DE = 10$, $EF = 13$, $DF = 16$, and $AC = 4x - 8$, find x .

THEOREM 4.3: THIRD ANGLES THEOREM

If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also congruent.

If $\angle A \cong \angle D$ and $\angle B \cong \angle E$,
then _____.



Example 3 Using the Third Angles Theorem

Find the value of x .

Solution

In the diagram, $\angle V \cong \angle Y$ and

$\angle U \cong \angle Z$. From the Third Angles

Theorem, you know that $\angle W \cong \angle X$.

So, $m\angle W = m\angle X$. From the Triangle Sum

Theorem, $m\angle W = 180^\circ - 54^\circ - 67^\circ = \underline{\hspace{1cm}}^\circ$.

$$m\angle W = m\angle X$$

Third Angles Theorem

$$\underline{\hspace{1cm}}^\circ = (4x - 5)^\circ$$

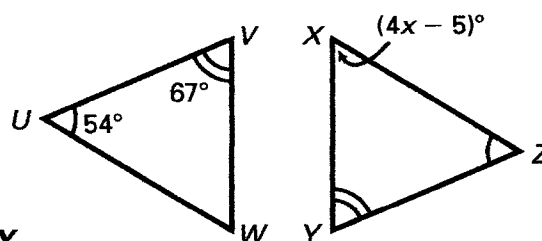
Substitute.

$$\underline{\hspace{1cm}} = 4x$$

Add $\underline{\hspace{1cm}}$ to each side.

$$\underline{\hspace{1cm}} = x$$

Divide each side by $\underline{\hspace{1cm}}$.



Look for:

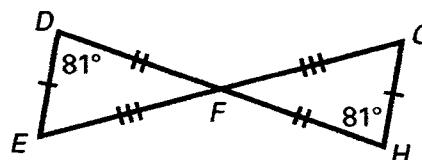
① vertical angles

② reflexive side or shared side

③ third angles

Example 4 Determining Whether Triangles are Congruent

Decide whether the triangles are congruent. Justify your reasoning.



Solution

Paragraph Proof From the diagram, you are given that all three pairs of corresponding sides are congruent.

$$DE \cong \underline{\hspace{1cm}}, \underline{\hspace{1cm}} \cong GF, DF \cong \underline{\hspace{1cm}}$$

Because $\angle D$ and $\angle H$ have the same measure, $\angle D \cong \angle H$. By the Vertical Angles Theorem, you know that _____.

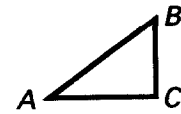
By the Third Angles Theorem, _____.

Answer So, all three pairs of corresponding sides and all three pairs of corresponding angles are _____. By the definition of congruent triangles, _____.

THEOREM 4.4: PROPERTIES OF CONGRUENT TRIANGLES

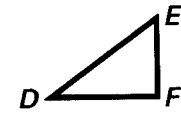
Reflexive Property of Congruent Triangles

Every triangle is congruent to _____.



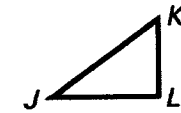
Symmetric Property of Congruent Triangles

If $\triangle ABC \cong \triangle DEF$, then _____.



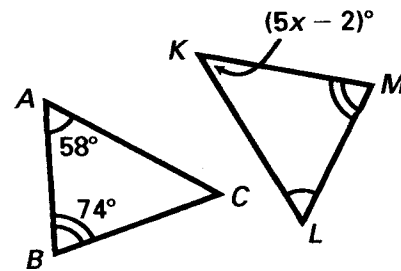
Transitive Property of Congruent Triangles

If $\triangle ABC \cong \triangle DEF$ and $\triangle DEF \cong \triangle JKL$, then _____.



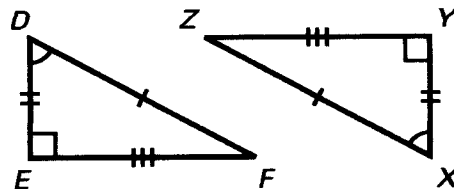
Checkpoint Complete the following exercises.

1. Find the value of x .



$$\angle K \cong \angle C$$

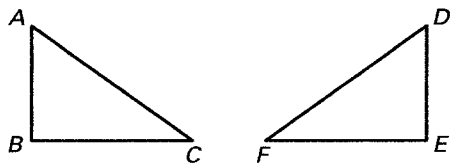
2. Decide whether the triangles are congruent. Justify your reasoning.



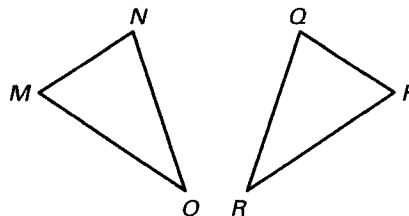
Practice A

For use with pages 202-210

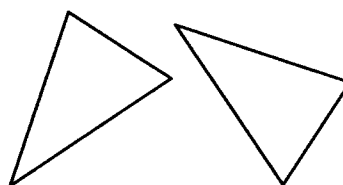
1. Given $\triangle ABC \cong \triangle DEF$, name three pairs of congruent sides.



2. Given $\triangle MNO \cong \triangle PQR$, name three pairs of congruent angles.

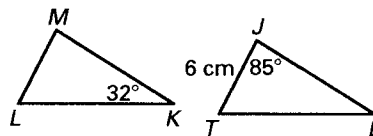


3. Copy the congruent triangles shown at the right. Then label the vertices of your triangles so that $\triangle RUV \cong \triangle TNF$. Identify all pairs of congruent corresponding angles and corresponding sides.



In the diagram, $\triangle MKL \cong \triangle JET$. Complete the statement.

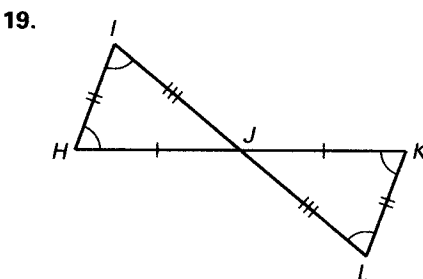
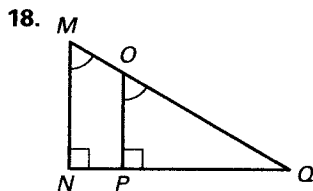
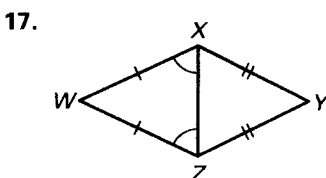
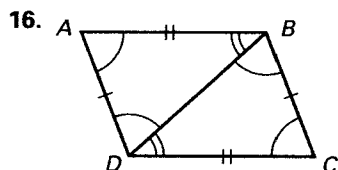
4. $\angle L \cong$ _____
 5. $\overline{MK} \cong$ _____
 6. $m\angle M =$ _____
 7. $m\angle T =$ _____
 8. $ML =$ _____
 9. $\triangle ETJ \cong$ _____



Complete this statement.

10. If $\triangle WRD \cong \triangle PLK$, then $\overline{WR} \cong$ _____.
 11. If $\triangle BGT \cong \triangle DSN$, then $\angle T \cong$ _____.
 12. If $\triangle SVP \cong \triangle MTQ$, then $\overline{PS} \cong$ _____.
 13. If $\triangle JCX \cong \triangle MWP$, then $\overline{XC} \cong$ _____.
 14. If $\triangle RHK \cong \triangle WVO$, then $\triangle KRH \cong$ _____.
 15. If $\triangle PMC \cong \triangle LDX$, then $\angle M \cong$ _____.

Identify any figures that can be proved congruent. Explain your reasoning. For those that can be proved congruent, write a congruence statement.



Standardized Test Practice

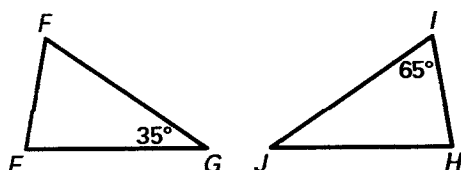
For use with pages 202–210

TEST TAKING STRATEGY One of the best ways to prepare for the SAT is to keep up with your regular studies and do your homework.

1. **Multiple Choice** If $\triangle ABC \cong \triangle XYZ$, which of the following statements below is *not* true?

- (A) $\angle B \cong \angle Y$ (B) $\overline{AB} \cong \overline{XY}$
 (C) $\angle CBA \cong \angle ZXY$ (D) $\overline{AC} \cong \overline{XZ}$
 (E) $\angle BAC \cong \angle YXZ$

2. **Multiple Choice** In the diagram, $\triangle EFG \cong \triangle HIJ$. What is the measure of $\angle H$?

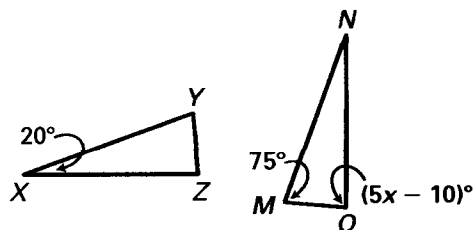


- (A) 35° (B) 65° (C) 70°
 (D) 80° (E) Cannot be determined

3. **Multiple Choice** In the diagram in Exercise 2, $EG = \underline{\hspace{1cm}}?$

- (A) HI (B) HJ (C) JI
 (D) FG (E) Cannot be determined

4. **Multiple Choice** Given $\angle X \cong \angle N$ and $\angle Z \cong \angle O$, find the value of x .

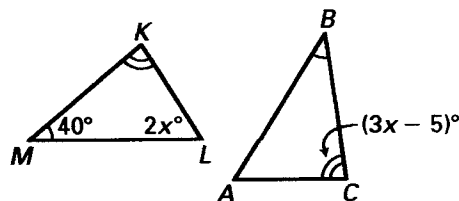


- (A) 19 (B) 38 (C) 95
 (D) 85 (E) 20

5. **Multiple Choice** Use the diagram in Exercise 4 to find $m\angle Z$.

- (A) 19° (B) 38° (C) 95°
 (D) 85° (E) 20°

6. **Multiple Choice** Given $\angle M \cong \angle B$ and $\angle K \cong \angle C$, find the value of x .

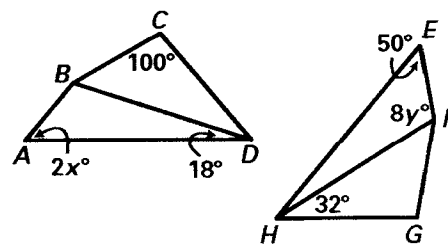


- (A) 43 (B) 40 (C) 82
 (D) 58 (E) 29

Quantitative Comparison In Exercises 7 and 8, use the given information to find the indicated value. Choose the statement below that is true about the given value.

- (A) The value in column A is greater.
 (B) The value in column B is greater.
 (C) The two values are equal.
 (D) The relationship cannot be determined from the given information.

Given: $ABCD \cong EFGH$



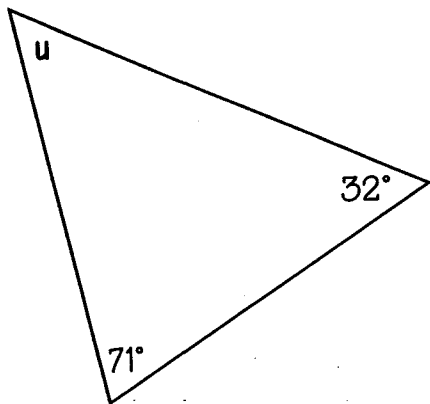
	Column A	Column B
7.	x	y
8.	$m\angle CBD$	$m\angle GHE$

SHOW ALL WORK ON NOTEBOOK PAPER

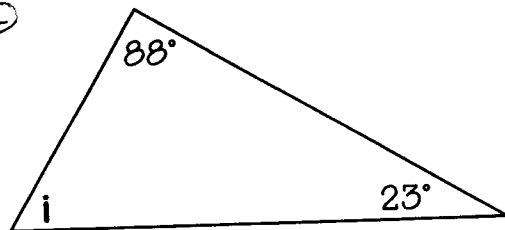
What is the main ingredient in Professor Crazy's dog biscuits?

Find the missing variables. To figure out the joke, place the letter of each problem above the answer on the line(s) below. Some blanks will go unfilled.

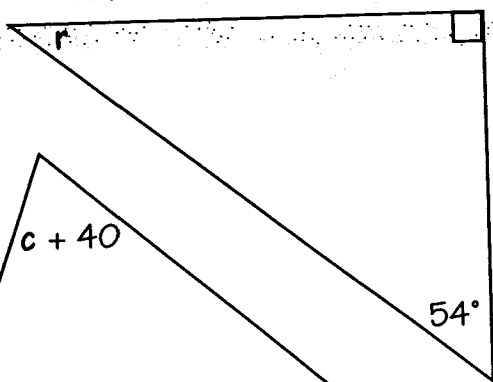
①



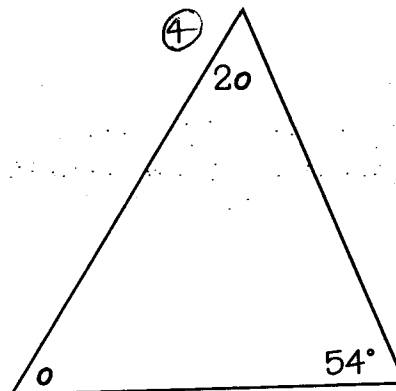
②



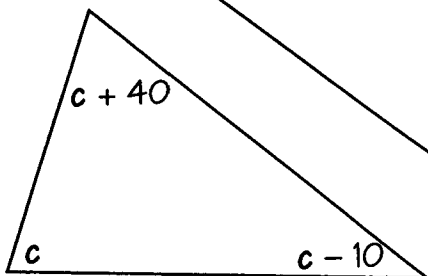
③



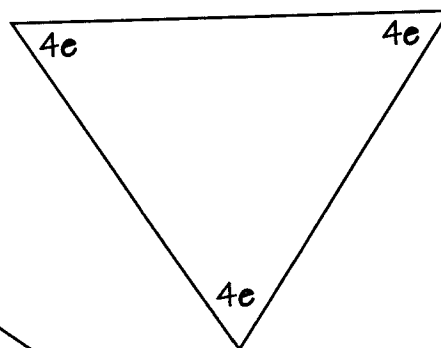
④



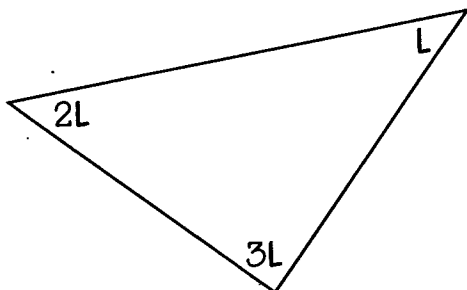
⑤



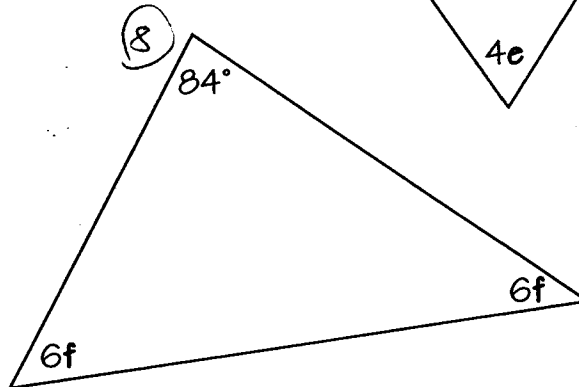
⑥



⑦



⑧



16 50 42 30 30 69 15 27 8 30 42 77 36 24 53 71

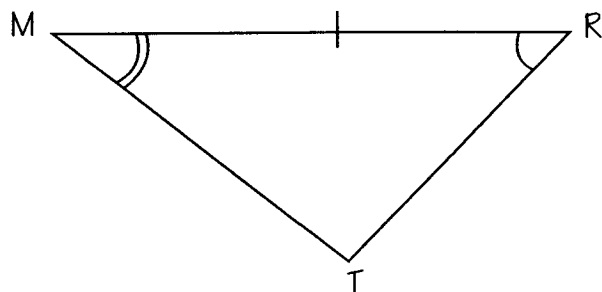
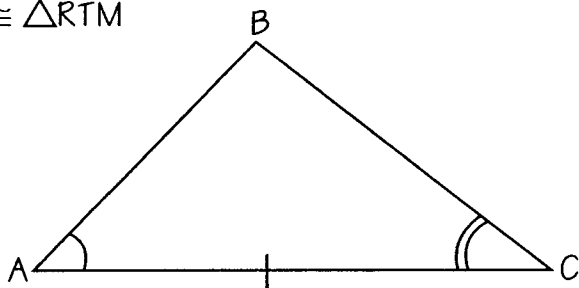
Triangles — Interior Angles

Joke #8

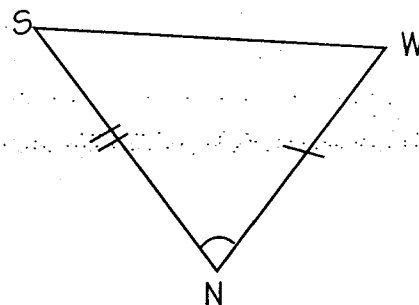
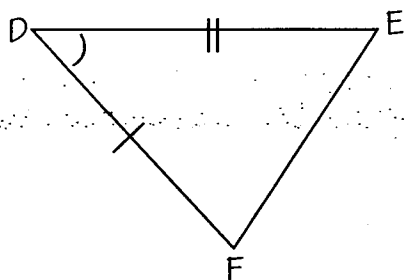
Play on Words: "How are ducks supposed to fly?"

Match congruent parts or shapes. Draw a line from the dot behind your answer in the first column to the dot in front of your answer in the second column. The line will contain a letter and a number Use the letter/number combinations to fill the blanks at the bottom.

$$\triangle ABC \cong \triangle RTM$$



$$\triangle DEF \cong \triangle NSW$$

 $\angle A \bullet 4$ $\overline{BC} \bullet$

4N.

 $\overrightarrow{FE} \cdot$ $\angle B \bullet$ $\overline{CA} \bullet$ $\triangle BAC \bullet$ $\Delta DFE \cdot$

∠W •

 $\Delta CBA \bullet$ $\triangle EDF \bullet$ $A \cdot \angle D$

- $\angle R$

 $\bullet \angle T$

• TM

• WS

$$r \cdot \angle F$$
$$R \bullet \overline{MR}$$

- Δ_{MTR}

- Δ_{TRM}

- Δ_{SNW}

- Δ_{NWS}

1	2	3	4	5	6	7	8	9	10	11
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Triangle Congruence Theorems

71

You look like twins... Are you? Let's have a little look at that there DNA shall we?

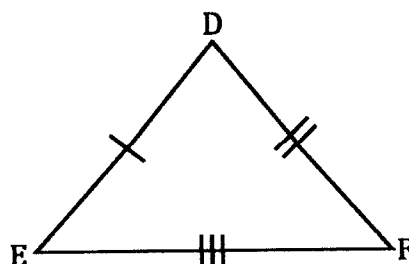
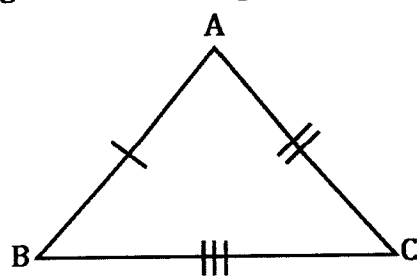
Remember back when we talked about what congruent meant? No? Okay, remember it means two shapes that are exactly the same size and shape...like identical twins. Okay, so two people run into each other walking down the sidewalk, they look exactly alike. Are they long lost identical twins, or just a freak coincidence of genetic mayhem? Well, there's a good way to find out... You guessed it! A DNA test will confirm if these two are twins or just scary.

The triangle congruence theorems are no different than a DNA test... Okay, well there is less blood and bodily fluids involved... but other than that they perform the same function... confirming identical triangle twins.

They are SSS, SAS, ASA, AAS, and HL (for right triangles only).

Huh? Okay, we will look at an example of each before we get into practicing.

SSS stands for Side, Side, Side. If three sides of one triangle are congruent to three sides of the other triangle then the triangles are congruent. Take a look...

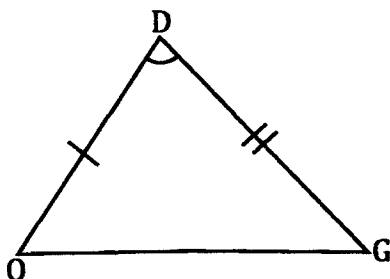
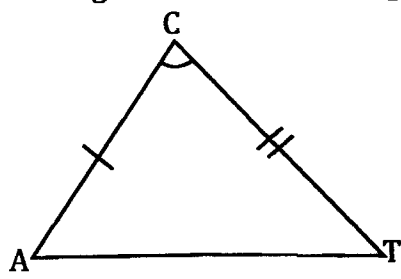


$\overline{AB} \cong \overline{DE}$ given
 $\overline{AC} \cong \overline{DF}$ given
 $\overline{BC} \cong \overline{EF}$ given

So $\triangle ABC \cong \triangle DEF$ by SSS.

We say "given" because they have been marked on the diagram like that info was "given" to us.

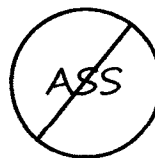
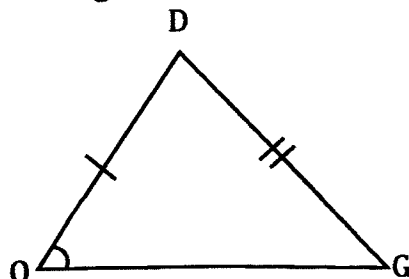
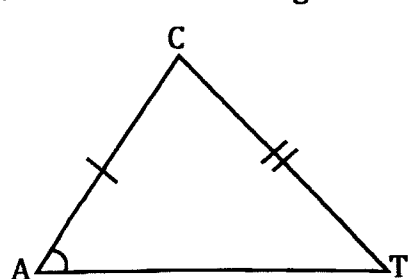
SAS stands for Side, Angle, Side. If two sides of a triangle and the included angle (means the one between the two sides) are congruent to the two sides, and an included angle of another triangle are congruent, then the triangles themselves are congruent. Take another look...



$\overline{AC} \cong \overline{OD}$ given
 $\angle C \cong \angle D$ given
 $\overline{CT} \cong \overline{DG}$ given

So $\triangle ACT \cong \triangle ODG$ by SAS.

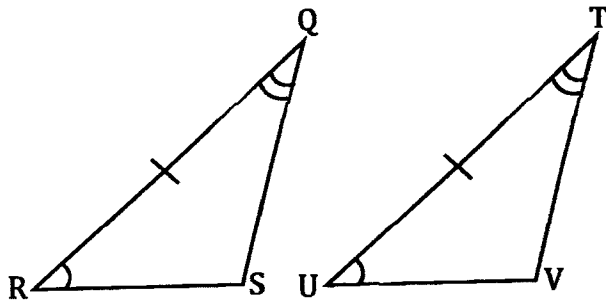
WATCH OUT! If the angles that are congruent are not between the two sides like this...



then it's not SAS, it's ASS. Don't say ASS! Don't use ASS! It is not a congruence property! Don't use SSA either, because it is just ASS backwards.

Annem....now that we have that cleared up...

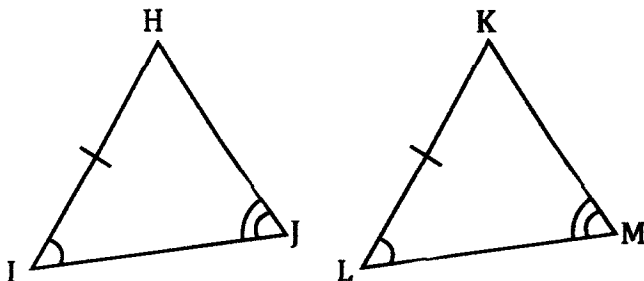
ASA stands for Angle, Side, Angle. If two angles of a triangle and the included side (means the one between the two angles) are congruent to the two angles and an included side of another triangle, then the triangles themselves are congruent. More looking for you...



$\angle R \cong \angle U$ given
 $\overline{RS} \cong \overline{UV}$ given
 $\angle S \cong \angle V$ given

So $\triangle QRS \cong \triangle TUV$ by ASA.

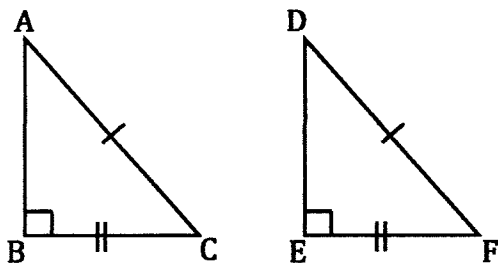
AAS stands for Angle, Angle, Side. It is really just a form of ASA. Think about it, If you have two angles in a triangle you can always find the third using the Triangle Sum Theorem. If you have AAS, you can find the third A, and get to ASA. Fortunately, we don't have to actually do this because it can be as hard as it sounds. Here is an example of AAS.



$\angle I \cong \angle L$ given
 $\angle J \cong \angle M$ given
 $\overline{HI} \cong \overline{KL}$ given

So $\triangle HIJ \cong \triangle KLM$ by AAS.

HL stands for Hypotenuse, Leg. What kind of triangle has a hypotenuse? A right triangle of course. This special congruence property is for right triangles only. If the hypotenuse and one leg of a right triangle is congruent to the hypotenuse and leg of another, then the right triangles are congruent. This is actually a shortcut for SSS because if you have two sides of a right triangle you can find the third using the Pythagorean theorem, thus arriving at SSS. Fortunately, you don't have to do that either.



$\overline{AC} \cong \overline{DF}$ given
 $\overline{BC} \cong \overline{EF}$ given

So $\triangle ABC \cong \triangle DEF$ by HL.

Make sure when you do this one that you actually have a hypotenuse and a leg and not two legs. (If you do have two legs the right angle will be between them giving you SAS. Trust me, you'll see.)

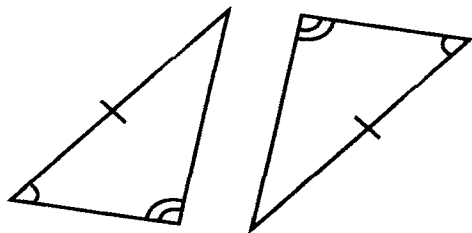
Great, these really don't get too hard except sometimes you have to come up with some parts that are congruent, like alternate interior angles, or shared sides. We will start with some simple examples. When we get to the little bit harder ones, I'll walk you through how to find the missing pieces.

To the twin mobile!...Okay, bad example.

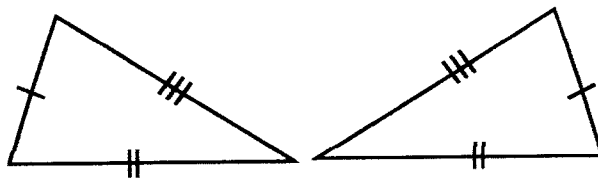
For each, state whether the triangles are congruent by A: SSS, B: SAS, C: ASA (AAS), or D: HL and bubble your answer below.

12

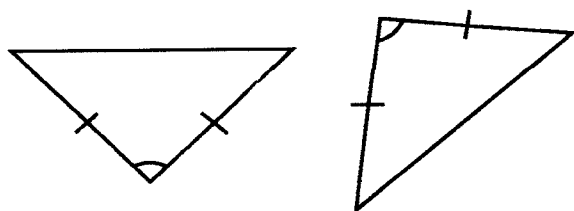
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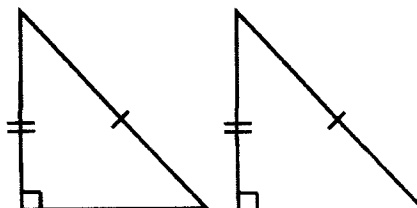
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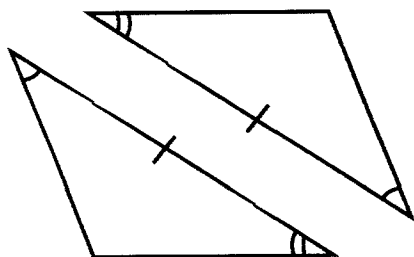
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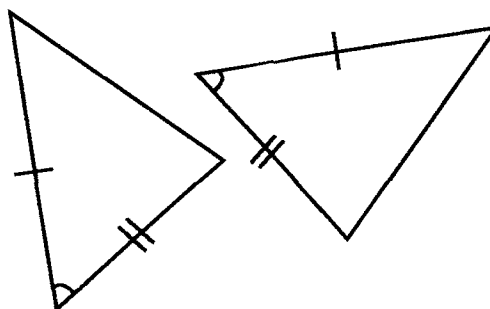
4.



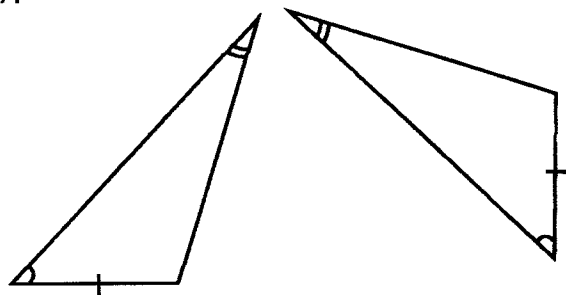
5.



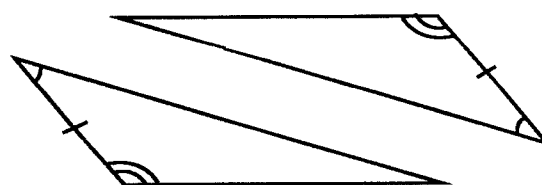
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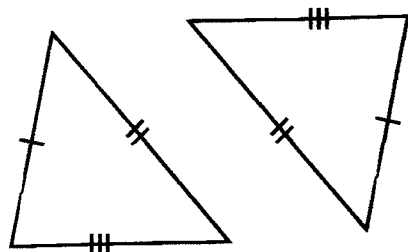
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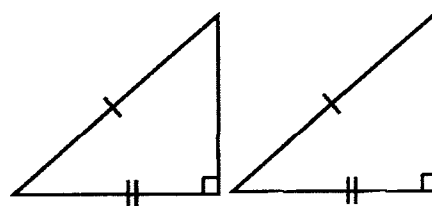
8.



9.



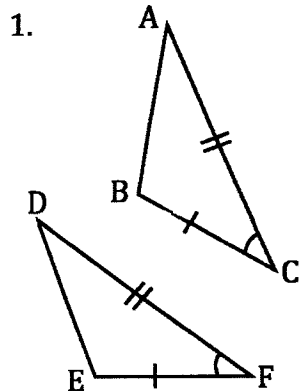
10.



Bubble the correct answer choice from each item above.

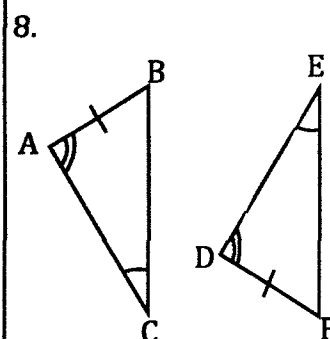
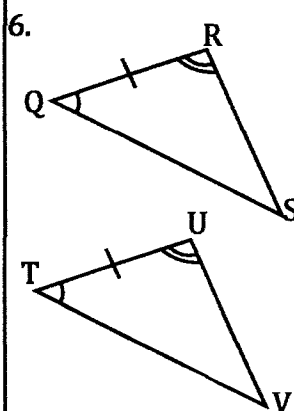
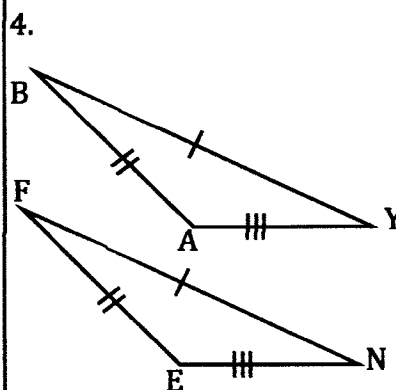
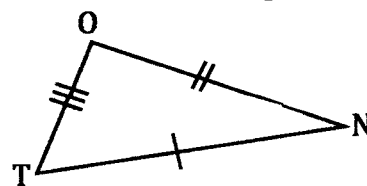
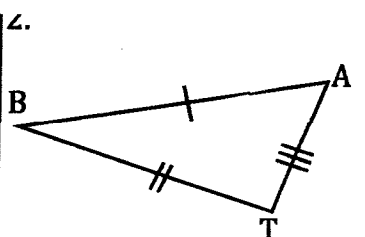
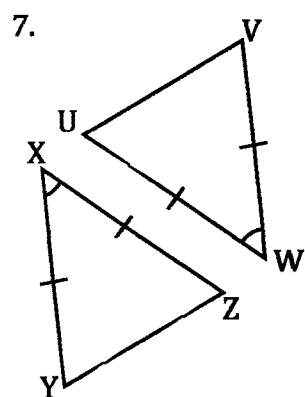
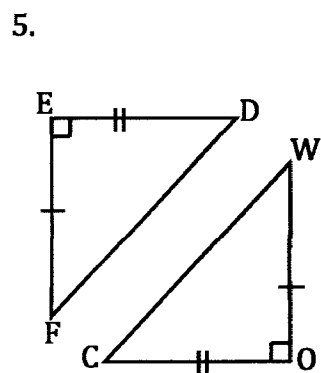
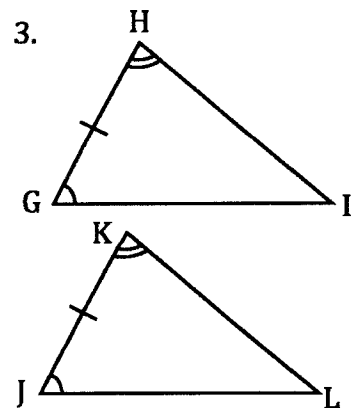
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<input type="radio"/> B.	<input type="radio"/> B.	<input type="radio"/> B.	<input type="radio"/> B.	<input type="radio"/> B.	<input type="radio"/> B.	<input type="radio"/> B.	<input type="radio"/> B.	<input type="radio"/> B.	<input type="radio"/> B.
<input type="radio"/> C.	<input type="radio"/> C.	<input type="radio"/> C.	<input type="radio"/> C.	<input type="radio"/> C.	<input type="radio"/> C.	<input type="radio"/> C.	<input type="radio"/> C.	<input type="radio"/> C.	<input type="radio"/> C.
<input type="radio"/> D.	<input type="radio"/> D.	<input type="radio"/> D.	<input type="radio"/> D.	<input type="radio"/> D.	<input type="radio"/> D.	<input type="radio"/> D.	<input type="radio"/> D.	<input type="radio"/> D.	<input type="radio"/> D.

List all the congruent parts. Then state whether the triangles are congruent by A: SSS, B: SAS, C: ASA (AAS), or D: HL and bubble your answer.



$\overline{BC} \cong \overline{EF}$ given
 $\angle C \cong \angle F$ given
 $\overline{AC} \cong \overline{DF}$ given

So $\triangle ABC \cong \triangle DEF$
 by SAS.

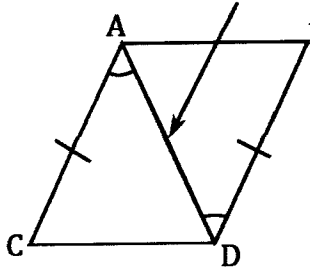


Bubble the correct answer choice from each item above.

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<input type="radio"/> B.	<input type="radio"/> B.	<input type="radio"/> B.	<input type="radio"/> B.	<input type="radio"/> B.	<input type="radio"/> B.	<input type="radio"/> B.	<input type="radio"/> B.
<input type="radio"/> C.	<input type="radio"/> C.	<input type="radio"/> C.	<input type="radio"/> C.	<input type="radio"/> C.	<input type="radio"/> C.	<input type="radio"/> C.	<input type="radio"/> C.
<input type="radio"/> D.	<input type="radio"/> D.	<input type="radio"/> D.	<input type="radio"/> D.	<input type="radio"/> D.	<input type="radio"/> D.	<input type="radio"/> D.	<input type="radio"/> D.

On some of these, you might need to find some hidden congruent parts. Also...some are not necessarily congruent. If they aren't, don't bubble anything. The first two are examples. Give it a go.

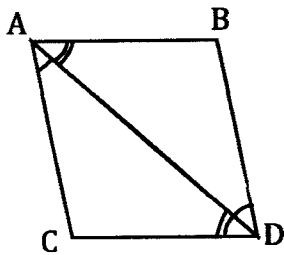
9. Shared Line



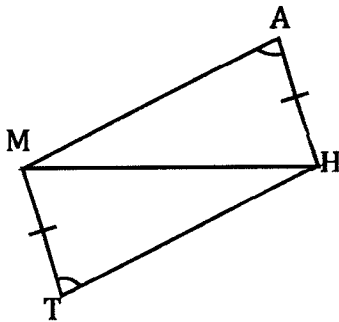
$\overline{CA} \cong \overline{BD}$ given
 $\angle CAD \cong \angle ADB$ given
 $\overline{AD} \cong \overline{AD}$ shared line

So $\triangle CAD \cong \triangle BDA$
 by SAS.

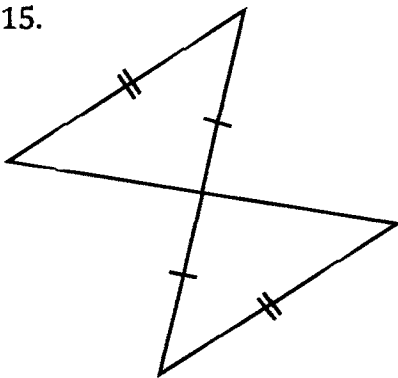
11.



13.

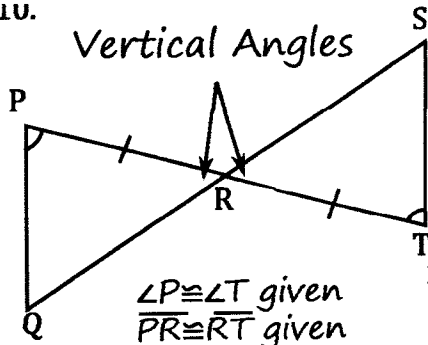


15.



10.

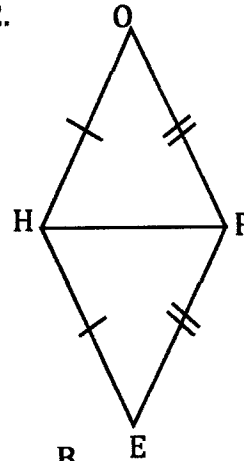
Vertical Angles



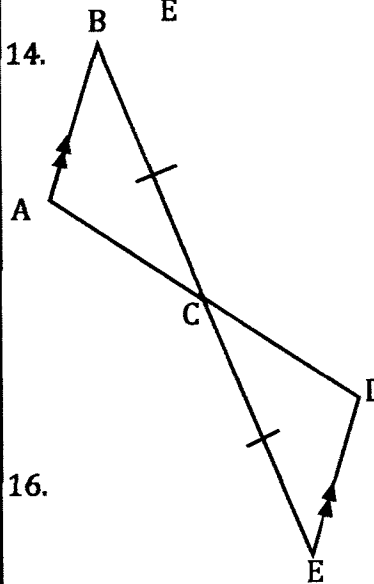
$\angle P \cong \angle T$ given
 $\overline{PR} \cong \overline{TR}$ given
 $\angle PRQ \cong \angle SRT$ vertical

So $\triangle PQR \cong \triangle TSR$
 by ASA.

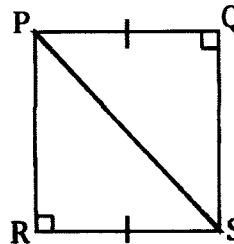
12.



14.

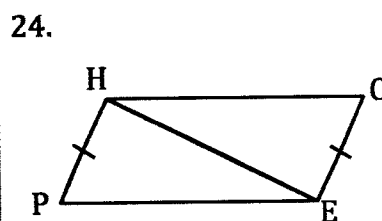
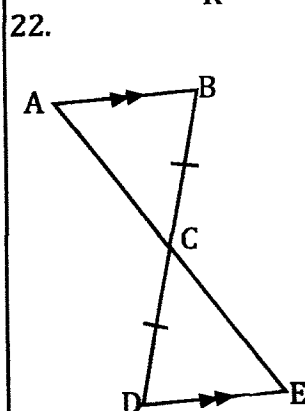
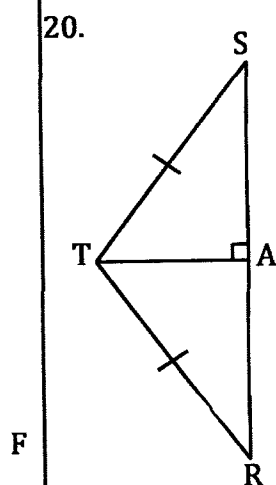
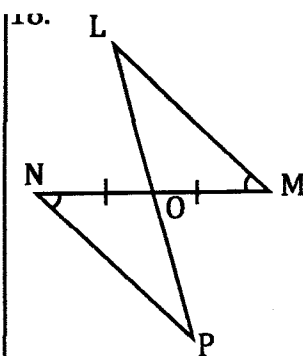
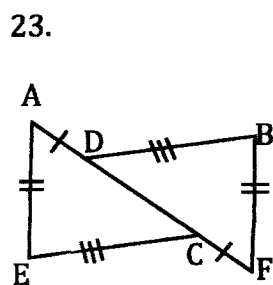
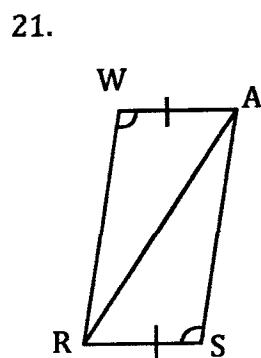
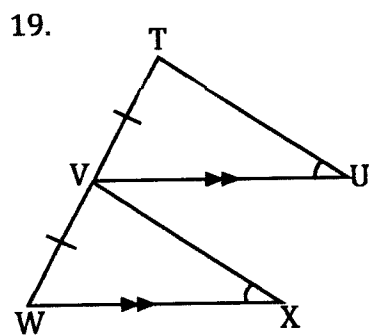
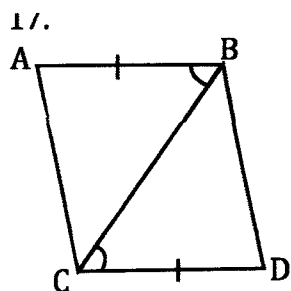


16.



Bubble the correct answer choice from each item above.

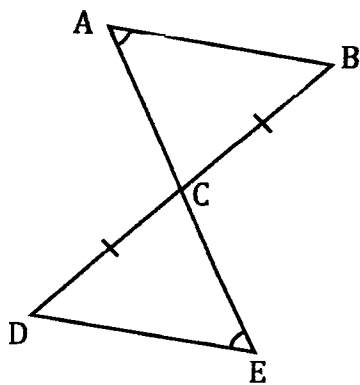
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<input type="radio"/> B.	<input type="radio"/> B.	<input type="radio"/> B.	<input type="radio"/> B.	<input type="radio"/> B.	<input type="radio"/> B.	<input type="radio"/> B.	<input type="radio"/> B.
<input type="radio"/> C.	<input type="radio"/> C.	<input type="radio"/> C.	<input type="radio"/> C.	<input type="radio"/> C.	<input type="radio"/> C.	<input type="radio"/> C.	<input type="radio"/> C.
<input type="radio"/> D.	<input type="radio"/> D.	<input type="radio"/> D.	<input type="radio"/> D.	<input type="radio"/> D.	<input type="radio"/> D.	<input type="radio"/> D.	<input type="radio"/> D.



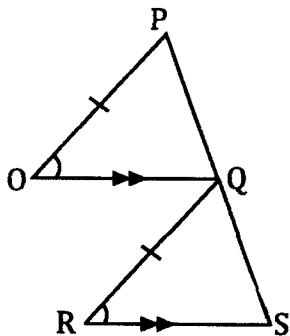
Bubble the correct answer choice from each item above.

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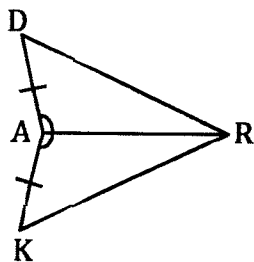
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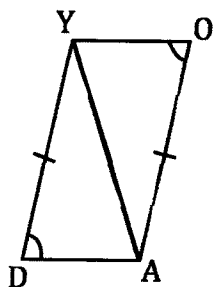
27.



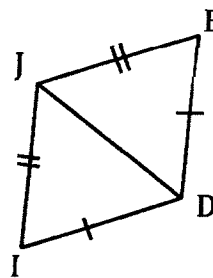
29.



31.

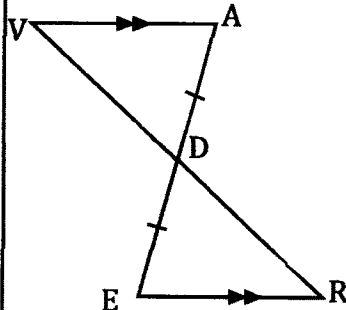


40.

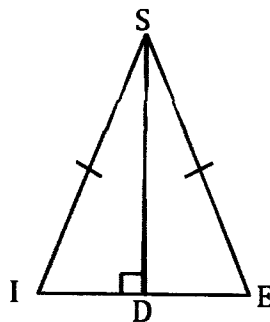


74

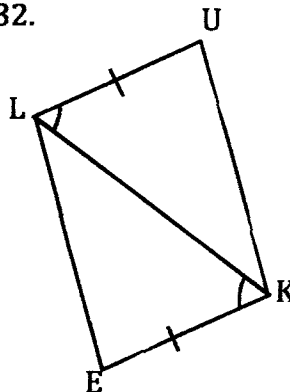
28.



30.



32.



Bubble the correct answer choice from each item above.

#25

- ☐ A.
☐ B.
☐ C.
☐ D.

#26

- ☐ A.
☐ B.
☐ C.
☐ D.

#27

- ☐ A.
☐ B.
☐ C.
☐ D.

#28

- ☐ A.
☐ B.
☐ C.
☐ D.

#29

- ☐ A.
☐ B.
☐ C.
☐ D.

#30

- ☐ A.
☐ B.
☐ C.
☐ D.

#31

- ☐ A.
☐ B.
☐ C.
☐ D.

#32

- ☐ A.
☐ B.
☐ C.
☐ D.

Skills Practice

4.4 Blue

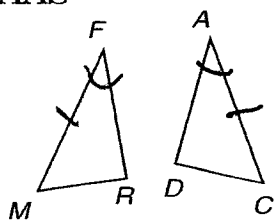
ASA and AAS

Write a congruence statement for each pair of triangles represented.

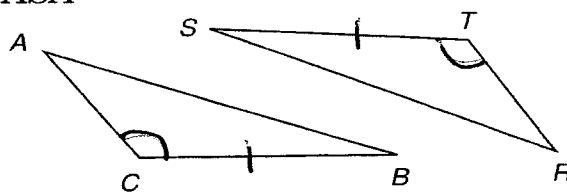
1. In $\triangle ABC$ and $\triangle ZXR$, $\angle C \cong \angle X$, $\angle A \cong \angle Z$, and $\overline{AB} \cong \overline{ZR}$.
2. In $\triangle DEF$ and $\triangle BGO$, $\angle D \cong \angle B$, $\angle E \cong \angle O$, and $\overline{DE} \cong \overline{BO}$.
3. In $\triangle TRI$ and $\triangle GAN$, $\angle T \cong \angle A$, $\overline{TI} \cong \overline{AG}$, and $\overline{TR} \cong \overline{AN}$.
4. In $\triangle ZIP$ and $\triangle LOS$, $\angle P \cong \angle O$, $\angle I \cong \angle L$, and $\overline{PI} \cong \overline{OL}$.

Name the additional congruent parts needed so that the triangles are congruent by the postulate or theorem indicated.

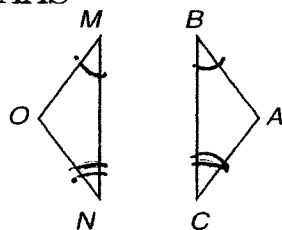
5. AAS



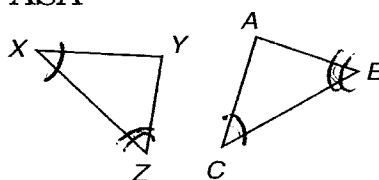
6. ASA



7. AAS

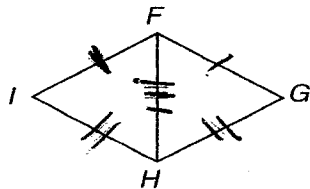


8. ASA

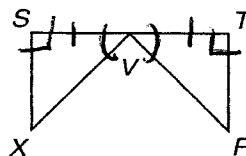


Determine whether each pair of triangles is congruent by SSS, SAS, ASA, or AAS. If it is not possible to prove that they are congruent, write not possible.

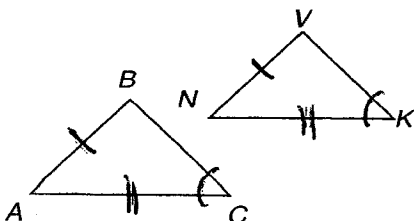
9.



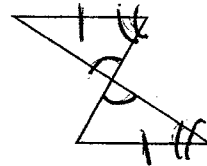
10.



11.

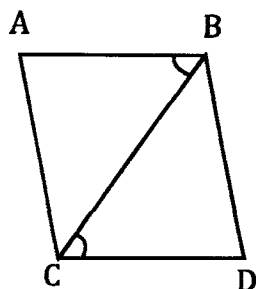


12.



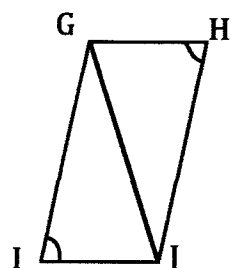
What additional information is needed to show the triangles are congruent by the given property?

1. SAS



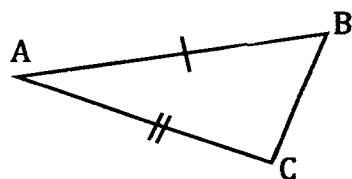
- A. $\overline{AC} \cong \overline{CD}$
- B. $\angle A \cong \angle D$
- C. $\overline{AB} \cong \overline{CD}$
- D. $\angle ABC \cong \angle DBC$

3. AAS

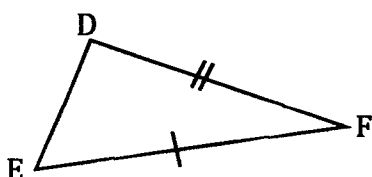


- A. $\angle IGJ \cong \angle HGI$
- B. $\overline{IG} \cong \overline{HJ}$
- C. $\angle IGJ \cong \angle HJG$
- D. $\angle HJG \cong \angle HGJ$

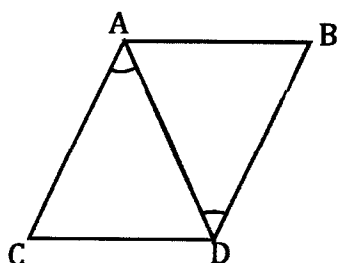
5. SAS



- A. $\angle A \cong \angle E$
- B. $\angle C \cong \angle D$
- C. $\angle B \cong \angle E$
- D. $\angle A \cong \angle F$

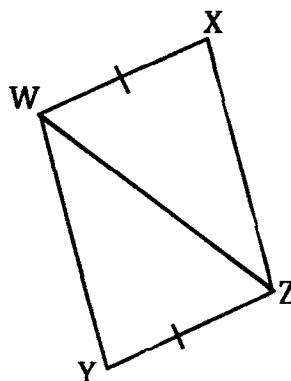


7. AAS



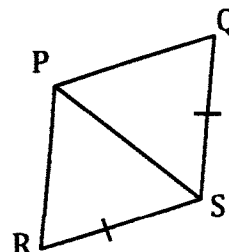
- A. $\angle C \cong \angle B$
- B. $\overline{CD} \cong \overline{AB}$
- C. $\angle BAD \cong \angle ABD$
- D. $\angle C \cong \angle BDA$

4. SSS



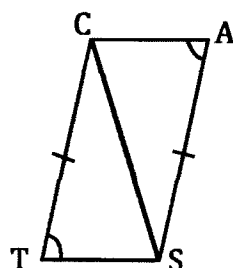
- A. $\overline{XZ} \cong \overline{YW}$
- B. $\angle X \cong \angle Y$
- C. $\overline{YZ} \cong \overline{XZ}$
- D. $\overline{WZ} \cong \overline{XZ}$

4. SAS



- A. $\angle QPS \cong \angle SQP$
- B. $\angle QSP \cong \angle RSP$
- C. $\angle QSP \cong \angle RPS$
- D. $\angle R \cong \angle Q$

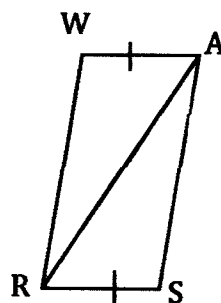
6. SAS



- A. Nothing
- B. $\angle TCS \cong \angle ASC$
- C. $\angle A \cong \angle T$
- D. $\angle TSC \cong \angle ACS$

E. $\overline{TS} \cong \overline{AC}$

8. SSS

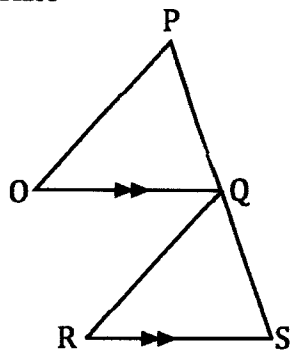


- A. $\overline{WR} \cong \overline{AR}$
- B. $\overline{AS} \cong \overline{AR}$
- C. $\overline{WR} \cong \overline{AS}$
- D. $\angle RAS \cong \angle ARW$

Bubble the correct answer choice from each item above.

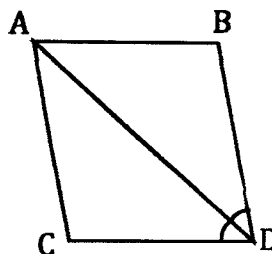
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<input type="radio"/> C.	<input type="radio"/> C.	<input type="radio"/> C.	<input type="radio"/> C.	<input type="radio"/> C.	<input type="radio"/> C.	<input type="radio"/> C.	<input type="radio"/> C.
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9. ASA



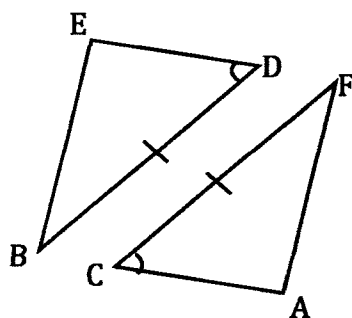
- A. $\angle PQO \cong \angle QSR$
- B. $\angle P \cong \angle R$
- C. $\angle O \cong \angle P$
- D. $\angle O \cong \angle R$

10. SAS



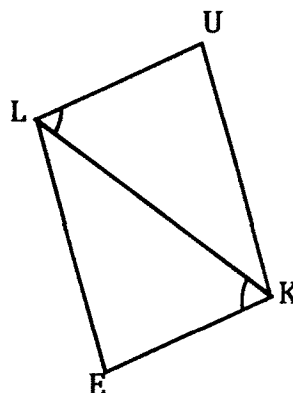
- A. $\overline{BD} \cong \overline{AC}$
- B. $\overline{BA} \cong \overline{CD}$
- C. $\angle B \cong \angle C$
- D. $\overline{CD} \cong \overline{BD}$

11. AAS



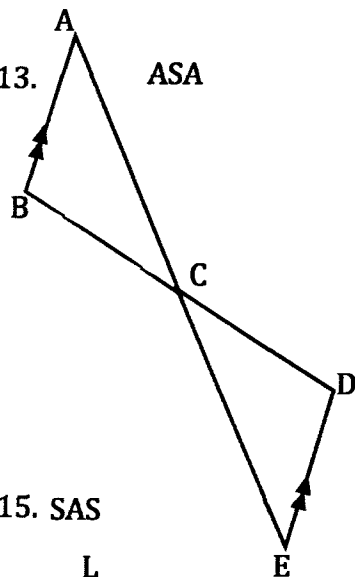
- A. $\angle F \cong \angle B$
- B. $\angle E \cong \angle A$
- C. $\angle E \cong \angle B$
- D. $\angle B \cong \angle A$

12. ASA



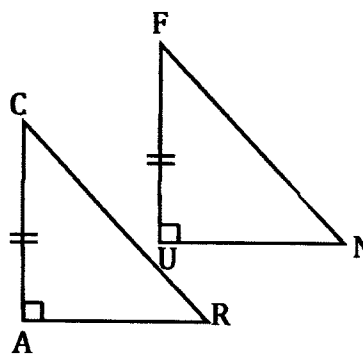
- A. $\angle U \cong \angle E$
- B. $\angle ELK \cong \angle UKL$
- C. $\angle U \cong \angle ELK$
- D. $\overline{LU} \cong \overline{EK}$

13. ASA



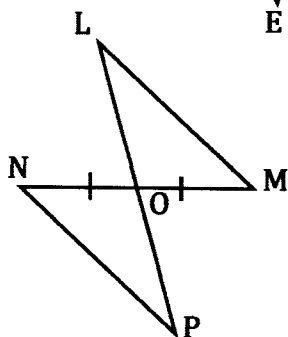
- A. $\overline{AC} \cong \overline{ED}$
- B. $\overline{AB} \cong \overline{CD}$
- C. $\overline{AC} \cong \overline{EC}$
- D. $\angle A \cong \angle E$

14. HL



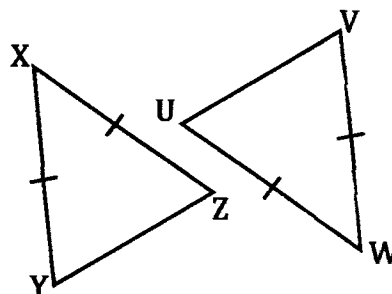
- A. $\overline{AR} \cong \overline{UN}$
- B. $\overline{CR} \cong \overline{UN}$
- C. $\overline{CR} \cong \overline{FU}$
- D. $\overline{CR} \cong \overline{FN}$

15. SAS



- A. NM bisects LP
- B. $\overline{NP} \cong \overline{LM}$
- C. $\overline{NP} \cong \overline{LO}$
- D. $\overline{LP} \cong \overline{MN}$

16. SSS



- A. $\angle X \cong \angle W$
- B. $\angle Y \cong \angle V$
- C. $\overline{YZ} \cong \overline{VU}$
- D. $\overline{YZ} \cong \overline{XZ}$

Bubble the correct answer choice from each item above.

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<input type="radio"/> C.	<input type="radio"/> C.	<input type="radio"/> C.	<input type="radio"/> C.	<input type="radio"/> C.	<input type="radio"/> C.	<input type="radio"/> C.	<input type="radio"/> C.
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Proofs Involving Congruent Triangles

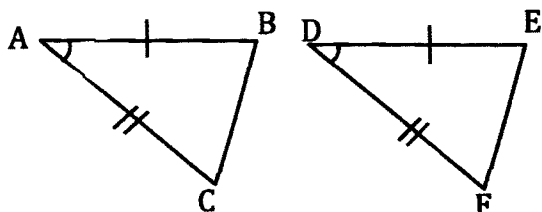
76

First, let's analyze some proofs.

This is easy! All you have to do is explain in plain English what is going on in the proofs. We'll look at some examples first.

AE. 1.

Given: $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, and $\angle A \cong \angle D$

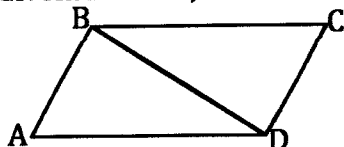


Prove: $\triangle ABC \cong \triangle DEF$

Statements	Reasons
1. $\overline{AB} \cong \overline{DE}$	1. Given
2. $\overline{AC} \cong \overline{DF}$	2. Given
3. $\angle A \cong \angle D$	3. Given
4. $\triangle ABC \cong \triangle DEF$	4. SAS

AE. 2.

Given: $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{CB}$

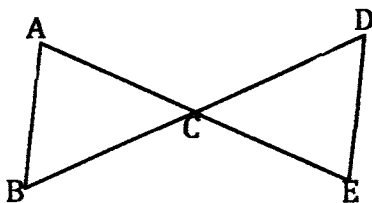


Prove: $\triangle ABD \cong \triangle CBD$

Statements	Reasons
1. $\overline{AB} \cong \overline{CD}$	1. Given
2. $\overline{AD} \cong \overline{CB}$	2. Given
3. $\overline{BD} \cong \overline{BD}$	3. Reflexive property
4. $\triangle ABD \cong \triangle CBD$	4. SSS

AE. 3.

Given: \overline{AE} Bisects \overline{BD} , $\angle B \cong \angle D$



Prove: $\triangle ABC \cong \triangle DEC$

Statements	Reasons
1. $\angle B \cong \angle D$	1. Given
2. \overline{AC} Bisects \overline{BD}	2. Given
3. $\overline{BC} \cong \overline{DC}$	3. Definition of Bisect
4. $\angle ACB \cong \angle DCE$	4. Vertical angles
5. $\triangle ABC \cong \triangle DEC$	5. ASA

Analysis:

Working backward we must ask the key question, "How can we show that two triangles are congruent?" The answer? A triangle congruence theorem like SSS, SAS, ASA, AAS or HL. This gives us B1: $\triangle ABC \cong \triangle DEC$, by some property, but which one? To find out, start working forward. Listing all of the given information gives us a pair of angles $\angle B$ and $\angle D$ sandwiched between a pair of congruent sides $\overline{AB} \cong \overline{DE}$ and $\overline{AC} \cong \overline{DF}$. So this means we have $\triangle ABC \cong \triangle DEF$ by the SAS theorem which is B2: and the proof is complete.

Analysis:

Working backward, we must ask the key question "How can we show that two triangles are congruent?" The answer? A triangle congruence theorem like SSS, SAS, ASA, AAS or HL. This gives us B1: $\triangle ABC \cong \triangle BCD$ by some property, but which one? Then start working forward. Listing all of the given information gives us two pairs of sides $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{CB}$, but this is not enough. We need another pair of sides or an angle between them. Looking now at the diagram we have $\overline{BD} \cong \overline{BD}$ as a shared line. So this brings us to say $\triangle ABC \cong \triangle BCD$ by SSS which is B1 and the proof is complete.

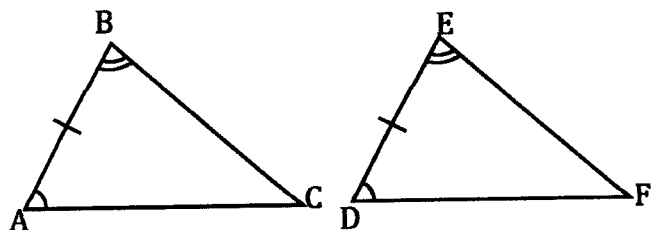
Analysis:

Working backward we must ask the key question, "How can we show that two triangles are congruent?" The answer? A triangle congruence theorem like SSS, SAS, ASA, AAS or HL. This gives us B1: $\triangle ABC \cong \triangle DEC$ by some property, but which one? Then start working forward. Listing all of the given information gives us a pair of angles $\angle B$ and $\angle D$, and \overline{BD} and \overline{AE} bisects \overline{BD} . If \overline{AE} bisects \overline{BD} then \overline{BD} is cut in half at C so $\overline{BC} \cong \overline{DC}$! This is not enough though. Looking at the diagram we see vertical angles $\angle ACB \cong \angle DCE$, which gives us $\triangle ABC \cong \triangle DEC$ by the property ASA. This is B1 and the proof is complete.

for these fill in any missing statements or reasons.

1.

Given: $\overline{AB} \cong \overline{DE}$, $\angle B \cong \angle E$, and $\angle A \cong \angle D$

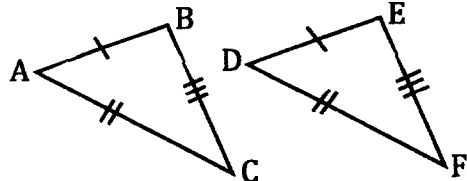


Prove: $\triangle ABC \cong \triangle DEF$

Statements	Reasons
1. $\overline{AB} \cong \overline{DE}$	1. Given
2.	2. Given
3. $\angle A \cong \angle D$	3.
4. $\triangle ABC \cong \triangle DEF$	4.

3.

Given: $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, and $\overline{BC} \cong \overline{EF}$

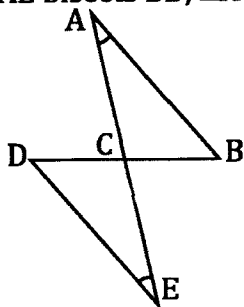


Prove: $\triangle ABC \cong \triangle DEF$

Statements	Reasons
1. $\overline{AB} \cong \overline{DE}$	1.
2.	2.
3.	3.
4.	4. SSS

5.

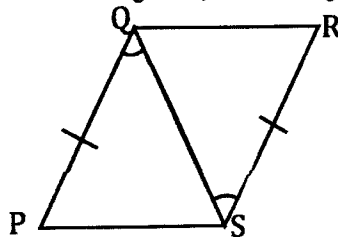
Given: \overline{AE} bisects \overline{BD} , $\angle A \cong \angle E$



Prove: $\triangle ABC \cong \triangle EDC$

Statements	Reasons
1. $\angle A \cong \angle E$	1.
2.	2. Given
3.	3. Definition of Bisect
4. $\angle ACB \cong \angle DCE$	4.
5. $\triangle ABC \cong \triangle EDC$	5.

2. Given: $\overline{PQ} \cong \overline{RS}$, and $\angle PQS \cong \angle RSQ$

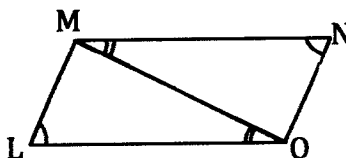


Prove: ~~$\triangle PQS \cong \triangle RSQ$~~

$\triangle PQS \cong \triangle RSQ$

Statements	Reasons
1.	1. Given
2.	2. Given
3. $\overline{QS} \cong \overline{QS}$	3.
4. $\triangle PQS \cong \triangle RSQ$	4.

4. Given: $\angle L \cong \angle N$, $\angle LOM \cong \angle NMO$

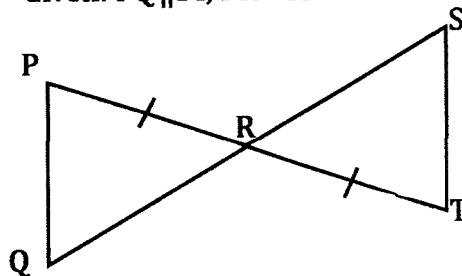


Prove: $\triangle LMO \cong \triangle NMO$

Statements	Reasons
1.	1.
2.	2. Given
3.	3. Reflexive Property
4. $\triangle LMO \cong \triangle NMO$	4.

6.

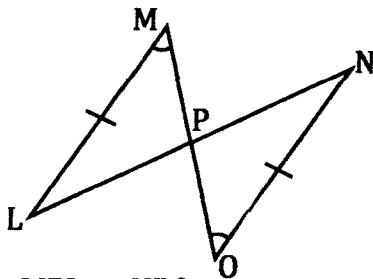
Given: $\overline{PQ} \parallel \overline{ST}$, $\overline{PR} \cong \overline{TR}$



Prove: $\triangle PQR \cong \triangle TSR$

Statements	Reasons
1. $\overline{PR} \cong \overline{TR}$	1.
2.	2. Given
3. $\angle P \cong \angle T$	3.
4. $\angle ACB \cong \angle DCE$	4.
5.	5. ASA

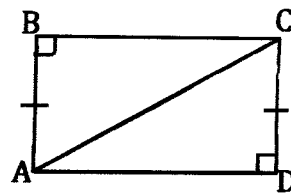
7. Given: $LM \cong NO$, and $\angle M \cong \angle O$



Prove: $\triangle MPL \cong \triangle NPO$

Statements	Reasons
1. $LM \cong NO$	1.
2.	2. Given
3.	3.
4.	4. AAS

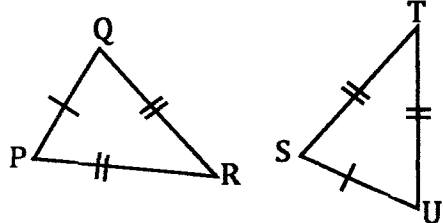
8. Given: $AB \cong DC$



Prove: $\triangle ABC \cong \triangle CDA$

Statements	Reasons
1.	1. Given
2. $AC \cong AC$	2.
3. $\triangle ABC \cong \triangle CDA$	3.

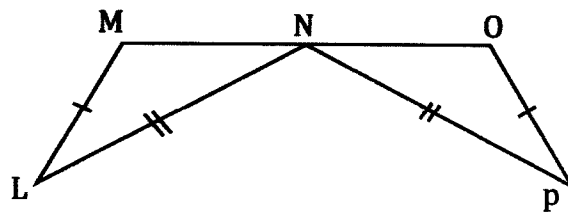
9. Given: $PQ \cong SU$, $QR \cong ST$, and $PR \cong TU$



Prove: $\triangle PQR \cong \triangle STU$

Statements	Reasons
1.	1. Given
2.	2. Given
3.	3.
4. $\triangle PQR \cong \triangle STU$	4.

10. Given: N is the midpoint of \overline{MO} , $LM \cong OP$, and $LN \cong PN$

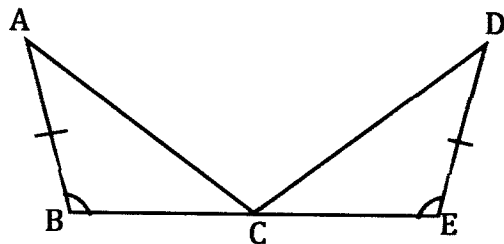


Prove: $\triangle LMN \cong \triangle PON$

Statements	Reasons
1. $LM \cong OP$	1. Given
2. $LN \cong PN$	2.
3. N is the Midpoint of \overline{MO}	3. Given
4.	4. Midpoint
5.	5. SSS

11.

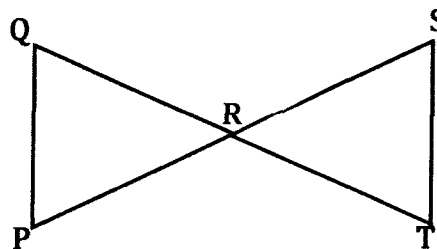
Given: C is the midpoint of \overline{BE} , $\angle B \cong \angle E$, and $AB \cong DE$



Prove: $\triangle ABC \cong \triangle DEC$

Statements	Reasons
1. $\angle B \cong \angle E$	1.
2. $AB \cong DE$	2.
3.	3. Given
4.	4. Midpoint
5. $\triangle ABC \cong \triangle DEC$	5. SAS

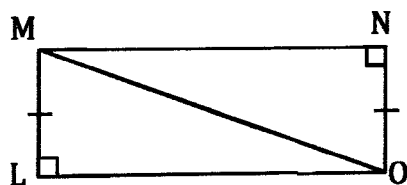
12. Given: \overline{QT} bisects \overline{SP} , \overline{SP} bisects \overline{QT}



Prove: $\triangle QRP \cong \triangle SRT$

Statements	Reasons
1. \overline{QT} bisects \overline{SP}	1. Given
2.	2. Given
3. $\overline{QR} \cong \overline{TR}$	3. Definition of Bisect
4. $\overline{PR} \cong \overline{SR}$	4.
5.	5. Vertical Angles
6. $\triangle QRP \cong \triangle SRT$	6.

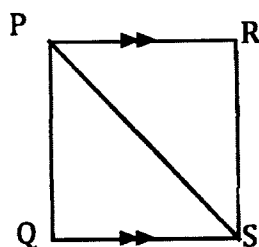
13. Given: $LM \cong NO$



Prove: $\triangle LMO \cong \triangle NOM$

Statements	Reasons
1. $\overline{LM} \cong \overline{NO}$	1.
2.	2.
3.	3.

15. Given: $\overline{PR} \parallel \overline{QS}$, $\angle QPS \cong \angle RSP$

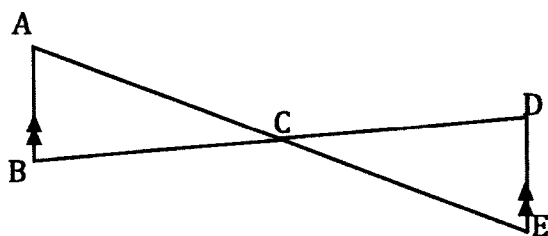


Prove: $\triangle PQS \cong \triangle SRP$

Statements	Reasons
1. $\overline{PR} \parallel \overline{QS}$	1.
2. $\angle QPS \cong \angle RSP$	2.
3. $\angle PSQ \cong \angle SPR$	3. Alternate Interior
4.	4. Reflexive Property
5. $\triangle PQS \cong \triangle SRP$	5.

17.

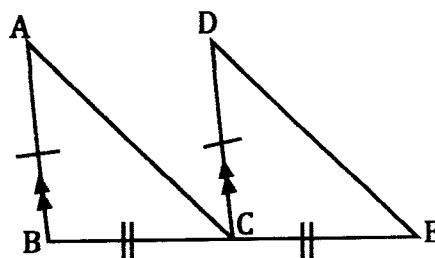
Given: \overline{AE} bisects \overline{BD} , $\overline{AB} \parallel \overline{DE}$



Prove: $\triangle ABC \cong \triangle DEC$

Statements	Reasons
1. \overline{AE} bisects \overline{BD}	1.
2.	2. Given
3. $\overline{BC} \cong \overline{DC}$	3.
4. $\angle ACB \cong \angle DCB$	4.
5.	5. Alternate Interior
6.	6. ASA

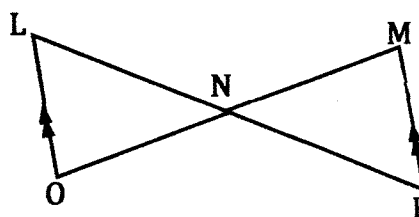
14. Given: $AB \cong DC$, $AB \parallel DC$, and $BC \cong CE$



Prove: $\triangle ABC \cong \triangle DCE$

Statements	Reasons
1. $\overline{AB} \cong \overline{DC}$	1. Given
2.	2. Given
3.	3. Given
4.	4. Corresponding Angles
5. $\triangle ABC \cong \triangle DCE$	5.

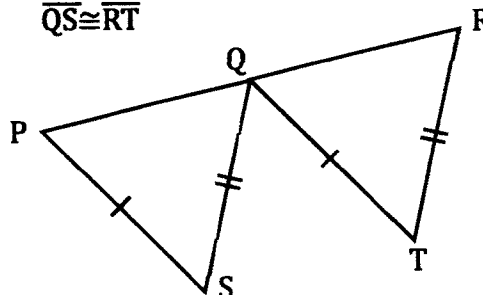
16. Given: \overline{LP} bisects \overline{MO} , $\overline{LO} \parallel \overline{MP}$



Prove: $\triangle LNO \cong \triangle MNP$

Statements	Reasons
1.	1. Given
2.	2. Given
3. $\overline{LN} \cong \overline{PN}$	3.
4.	4. Alternate Interior
5.	5. Vertical Angles
6.	6. ASA

18. Given: Q is the midpoint of \overline{PR} , $\overline{PS} \cong \overline{QT}$ and $\overline{QS} \cong \overline{RT}$



Prove: $\triangle PQS \cong \triangle RQT$

Statements	Reasons
1.	1. Given
2.	2. Given
3. $\overline{QS} \cong \overline{RT}$	3.
4.	4. Midpoint
5. $\triangle PQS \cong \triangle RQT$	5.

4.6

Isosceles, Equilateral, and Right Triangles

- Goals**
- Use properties of isosceles and equilateral triangles.
 - Use properties of right triangles.

VOCABULARY

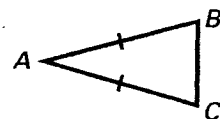
Base angles

Vertex angle

THEOREM 4.6: BASE ANGLES THEOREM

If two sides of a triangle are congruent, then the angles opposite them are congruent.

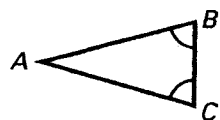
If $\overline{AB} \cong \overline{AC}$, then $\angle B \cong \angle C$.



THEOREM 4.7: CONVERSE OF THE BASE ANGLES THEOREM

If two angles of a triangle are congruent, then the sides opposite them are congruent.

If $\angle B \cong \angle C$, then $\overline{AB} \cong \overline{AC}$.



COROLLARY TO THEOREM 4.6

If a triangle is equilateral, then it is equiangular.

COROLLARY TO THEOREM 4.7

If a triangle is equiangular, then it is equilateral.

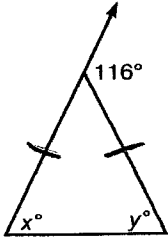
Practice

Isosceles Triangles

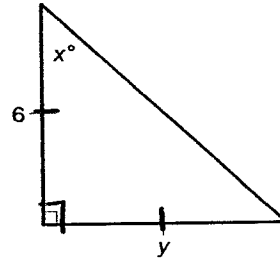
For each triangle, find the values of the variables.

Ans

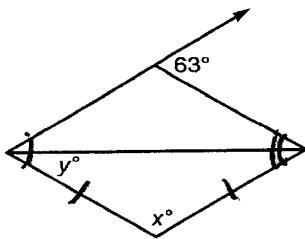
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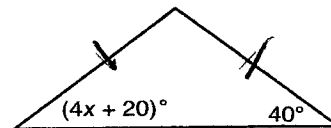
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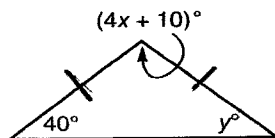
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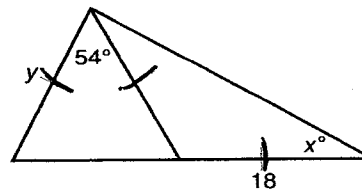
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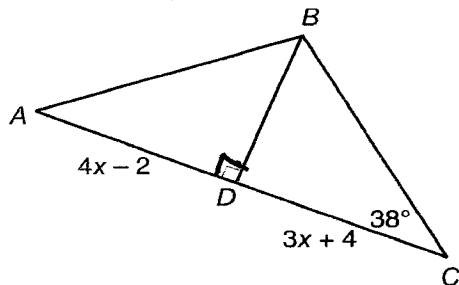
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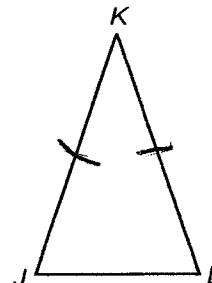
6.



7. In $\triangle ABC$, $m\angle A = m\angle C$ and $m\angle C = 38$.
Find $m\angle A$, AD , and AC .



8. In $\triangle JKL$, $\overline{JK} \cong \overline{KL}$. If $\angle J = 4x - 8$ and $\angle L = 3x + 15$, find $m\angle J$ and $m\angle L$.

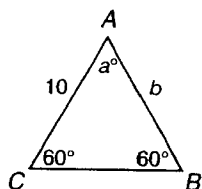


Skills Practice

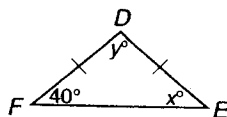
Isosceles Triangles

Find the values of the variables for each triangle.

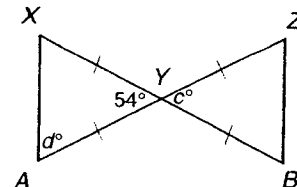
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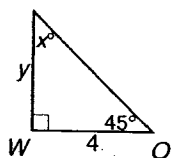
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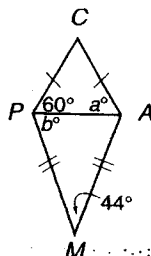
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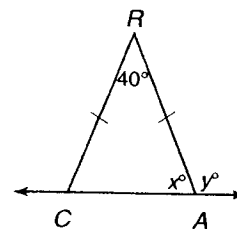
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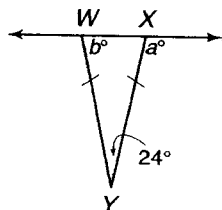
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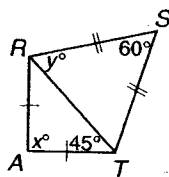
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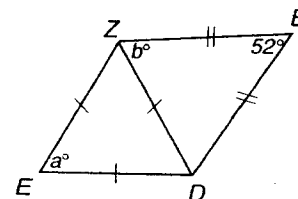
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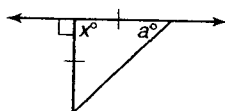
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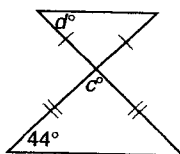
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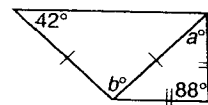
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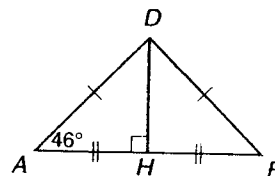
11.



12.



Use the figure at the right.

13. In $\triangle ADF$, if $AD = x + 6$ and $DF = 3x - 10$, what is AD ?14. In $\triangle ADH$, if $m\angle ADH = 2x - 4$, find the value of x .15. If $AH = 5x - 1$ and $FH = 3x + 21$, what is AH ?16. In $\triangle ADF$, what is $m\angle ADF$?

Study Guide

Isosceles Triangles

Remember that two sides of an isosceles triangle are congruent. Two important theorems about isosceles triangles are as follows.

If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

Example: Find the value of x .

Since $\overline{AB} \cong \overline{BC}$, the angles opposite \overline{AB} and \overline{BC} are congruent. So $m\angle A = m\angle C$.

Therefore, $3x - 10 = 2x + 6$.

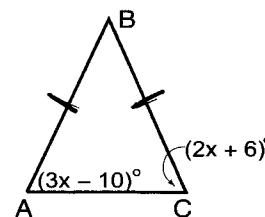
$$3x - 10 = 2x + 6$$

$$3x - 10 + 10 = 2x + 6 + 10 \quad \text{Add 10 to each side.}$$

$$3x = 2x + 16$$

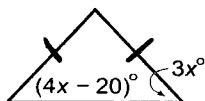
$$3x - 2x = 2x + 16 - 2x \quad \text{Subtract 2x from each side.}$$

$$x = 16$$

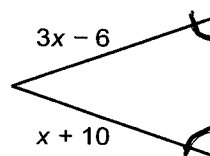


Find the value of x .

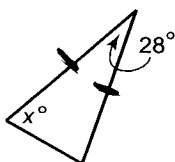
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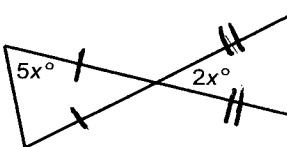
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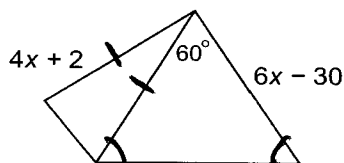
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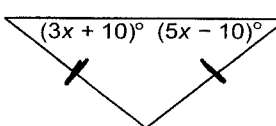
4.



5.



6.



Proofs Involving CPCTC

How to fix your car...

80

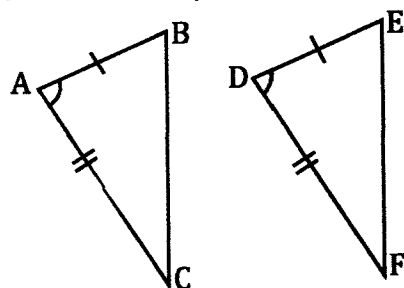
Okay, remember that to use CPCTC (Corresponding Parts of Congruent Triangles are Congruent), it's like saying that the carburetor from a '57 Chevy will be the same as the carburetor from another '57 Chevy. But, if you have two carburetors from two unknown cars, who knows if they are same or not? Okay, maybe a experienced mechanic could tell, but not me.

So remember... BEFORE YOU USE CPCTC YOU MUST PROVE THAT THE TRIANGLES IN QUESTION ARE CONGRUENT FIRST!!!

Let's analyze a couple of these, and then we will get to practicing...

Ex. 1.

Given: $\overline{AB} \cong \overline{DE}$, $\angle A \cong \angle D$, and $\overline{AC} \cong \overline{DF}$



Prove: $\angle C \cong \angle F$

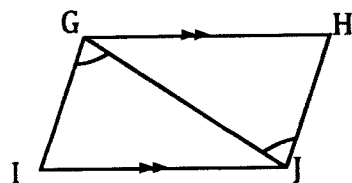
Statements	Reasons
1. $\overline{AB} \cong \overline{DE}$	1. Given
2. $\angle A \cong \angle D$	2. Given
3. $\overline{AC} \cong \overline{DF}$	3. Given
B2: 4. $\triangle ABC \cong \triangle DEF$	4. SAS
B1: 5. $\angle C \cong \angle F$	5. CPCTC

Analysis:

Working backwards, the statement "Prove: $\angle C \cong \angle F$ " begs the key question, "How can we show two segments from two different triangles are congruent?" The answer? CPCTC. This means that if we can prove the triangles are congruent then $\angle C \cong \angle F$ because they are corresponding parts of congruent triangles. So B1: $\angle C \cong \angle F$ by CPCTC. This then begs the second key question, "How can I show two triangles are congruent?" This means B2 will be $\triangle ABC \cong \triangle DEF$ by some congruence property. Which one? Now we work forward and see we have. $\overline{AB} \cong \overline{DE}$, $\angle A \cong \angle D$, and $\overline{AC} \cong \overline{DF}$ which are all given, This means $\triangle ABC \cong \triangle DEF$, which is B2, and the proof is complete.

Ex. 2.

Given: $\overline{GH} \parallel \overline{IJ}$, $\angle IGJ \cong \angle HJG$



Prove: $\overline{IG} \cong \overline{HJ}$

Statements	Reasons
1. $\overline{GH} \parallel \overline{IJ}$	1. Given
2. $\angle IGJ \cong \angle HJG$	2. Given
3. $\angle HGJ \cong \angle IJG$	3. Alternate Interior
4. $\overline{GJ} \cong \overline{GJ}$	4. Reflexive Property
B2: 5. $\triangle IGJ \cong \triangle HJG$	5. ASA
B1: 6. $\overline{IG} \cong \overline{HJ}$	6. CPCTC

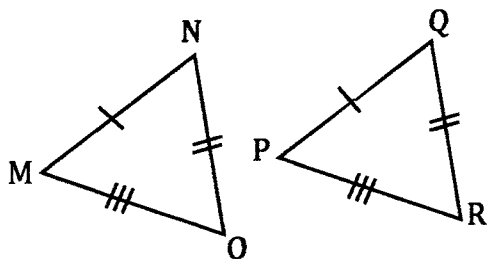
Analysis:

Working backwards, the statement "Prove: $\overline{IG} \cong \overline{HJ}$ " begs the key question, "How can we show two segments from two different triangles are congruent?" The answer? CPCTC. This means that if we can prove the triangles are congruent then $\overline{IG} \cong \overline{HJ}$ because they are corresponding parts of congruent triangles. So B1 $\overline{IG} \cong \overline{HJ}$ by CPCTC. This then begs the second key question, "How can I show two triangles are congruent?" This means B2 will be $\triangle IGJ \cong \triangle HJG$ by some congruence property. Which one? Now we work forward and see we have $\overline{GH} \parallel \overline{IJ}$, and $\angle IGJ \cong \angle HJG$, Given. We know when we see parallel lines we look for alternate interior or corresponding angles. We have alternate interior angles $\angle HGJ \cong \angle IJG$. We also have \overline{GJ} as a shared side. This means $\triangle IGJ \cong \triangle HJG$ by ASA, which is B2, and the proof is complete.

write an analysis or each proof below.

1. Given: $\overline{MN} \cong \overline{PQ}$, $\overline{NO} \cong \overline{QR}$, and $\overline{OM} \cong \overline{RP}$

Analysis:

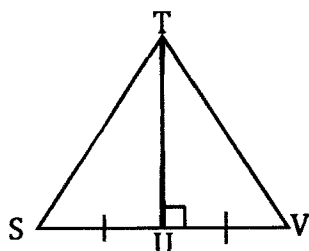


Prove: $\angle M \cong \angle P$

Statements	Reasons
1. $\overline{MN} \cong \overline{PQ}$	1. Given
2. $\overline{NO} \cong \overline{QR}$	2. Given
3. $\overline{OM} \cong \overline{RP}$	3. Given
B2: 4. $\triangle MNO \cong \triangle PQR$	4. SSS
B1: 5. $\angle M \cong \angle P$	5. CPCTC

2. Given: $\overline{SU} \cong \overline{UV}$

Analysis:

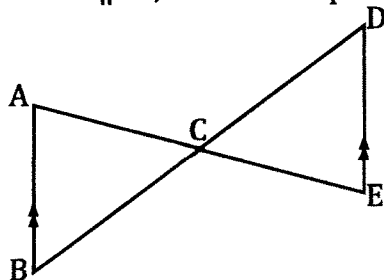


Prove: $\overline{ST} \cong \overline{VT}$

Statements	Reasons
1. $\overline{SU} \cong \overline{UV}$	1. Given
2. $\overline{TU} \cong \overline{TU}$	2. Reflexive Properties
B2: 3. $\triangle STU \cong \triangle VUT$	3. HL
B1: 4. $\overline{ST} \cong \overline{VT}$	4. CPCTC

3. Given: $\overline{AB} \parallel \overline{DE}$, C is the midpoint of \overline{AE}

Analysis:

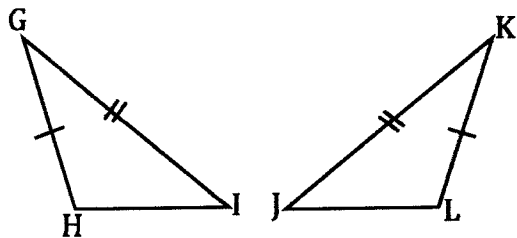


Prove: $\overline{BC} \cong \overline{DC}$

Statements	Reasons
1. $\overline{AB} \cong \overline{DE}$	1. Given
2. C is the midpoint of \overline{AE}	2. Given
3. $\angle BAC \cong \angle DEC$	3. Alternate Interior
4. $\overline{AC} \cong \overline{EC}$	4. Def. of Midpoint
5. $\angle ACB \cong \angle DCE$	5. Vertical Angles
B2: 6. $\triangle ABC \cong \triangle DEC$	6. ASA
B1: 7. $\overline{BC} \cong \overline{DC}$	7. CPCTC

Fill in the missing information in each proof.

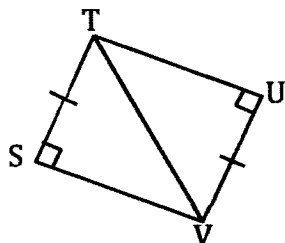
4. Given: $\overline{GH} \cong \overline{KL}$, $\angle G \cong \angle K$, and $\overline{GI} \cong \overline{KJ}$



Prove: $\overline{HI} \cong \overline{LJ}$

Statements	Reasons
1. $\overline{GH} \cong \overline{KL}$	1. Given
2.	2. Given
3. $\overline{GI} \cong \overline{KJ}$	3.
4.	4. SAS
5. $\overline{HI} \cong \overline{LJ}$	5.

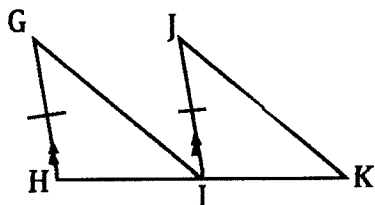
6. Given: $\overline{ST} \cong \overline{VU}$



Prove: $\angle SVT \cong \angle UTV$

Statements	Reasons
1.	1. Given
2.	2. Reflexive Property
3.	3. HL
4. $\angle SVT \cong \angle UTV$	4.

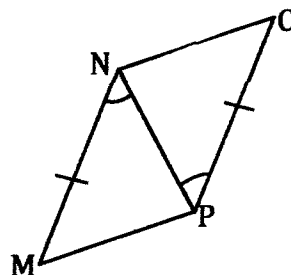
8. Given: $\overline{GH} \parallel \overline{JI}$, I is the midpoint of HK and $\overline{GH} \cong \overline{JI}$



Prove: $\angle G \cong \angle J$

Statements	Reasons
1. $\overline{GH} \parallel \overline{JI}$	1.
2. I is the midpoint of \overline{HK}	2.
3.	3. Given
4. $\overline{HI} \cong \overline{KI}$	4.
5.	5. Corresponding
6.	6. SAS
7. $\angle G \cong \angle J$	7.

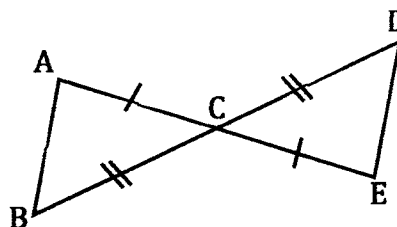
5. Given: $\angle MNP \cong \angle OPN$, and $\overline{MN} \cong \overline{OP}$



Prove: $\overline{MP} \cong \overline{NO}$

Statements	Reasons
1.	1. Given
2. $\overline{MN} \cong \overline{OP}$	2.
3. $\overline{NP} \cong \overline{NP}$	3.
4. $\triangle MNP \cong \triangle OPN$	4.
5.	5. CPCTC

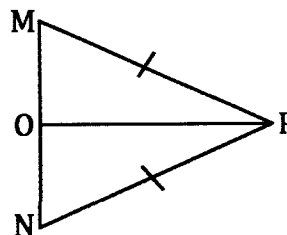
7. Given: $\overline{AC} \cong \overline{CE}$, $\overline{DC} \cong \overline{BC}$



Prove: $\angle B \cong \angle D$

Statements	Reasons
1.	1.
2.	2. Given
3. $\angle ACB \cong \angle DCE$	3.
4. $\triangle ABC \cong \triangle DEC$	4.
5. $\angle B \cong \angle D$	5.

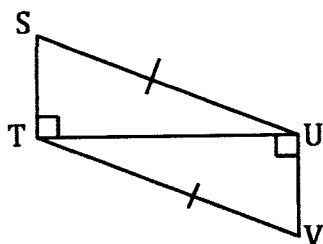
9. Given: $\overline{MP} \cong \overline{NP}$, $\overline{MN} \perp \overline{OP}$



Prove: $\overline{MO} \cong \overline{NO}$

Statements	Reasons
1.	1. Given
2. $\overline{MN} \perp \overline{OP}$	2.
3. $\overline{OP} \cong \overline{OP}$	3.
4. $\triangle MOP \cong \triangle NOP$	4.
5.	5.

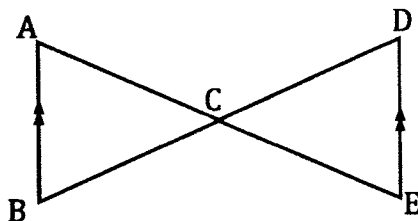
10. Given: $SU \cong VT$



Prove: $\overline{ST} \cong \overline{UV}$

Statements	Reasons
1. $\overline{SU} \cong \overline{VT}$	1.
2.	2.
3.	3. HL
4.	4. CPCTC

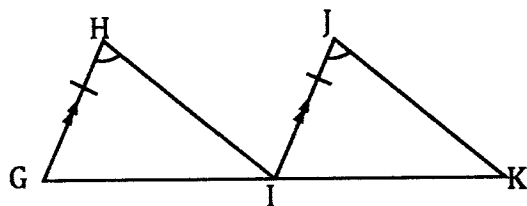
11. Given: $AB \parallel DE$, AE bisects BD



Prove: $\overline{AC} \cong \overline{EC}$

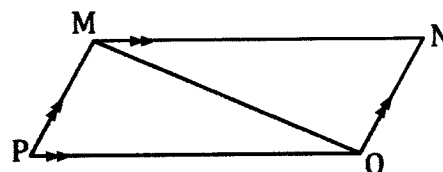
Statements	Reasons
1.	1.
2.	2. Given
3. $\angle ABC \cong \angle EDC$	3.
4. $\angle ACB \cong \angle DCE$	4.
5.	5. Def of Bisect
6. $\triangle ABC \cong \triangle EDC$	6.
7.	7.

12. Given: $GH \parallel IJ$, $\angle H \cong \angle J$ and $GH \cong IJ$



Prove: $\angle GIH \cong \angle IKJ$

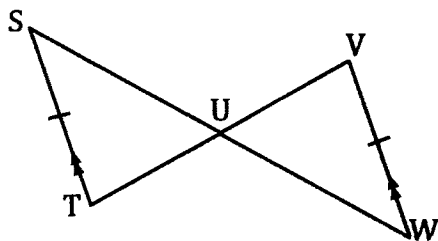
Statements	Reasons
1.	1. Given
2. $\angle H \cong \angle J$	2.
3.	3.
4.	4. Alternate Interior
5.	5.
6.	6. CPCTC



Prove: $PM \cong ON$

Statements	Reasons
1. $PM \parallel ON$	1.
2.	2. Given
3. $\angle PMO \cong \angle NOP$	3.
4.	4. Alternate Interior
5. $\overline{MO} \cong \overline{MO}$	5.
6.	6. ASA
7.	7.

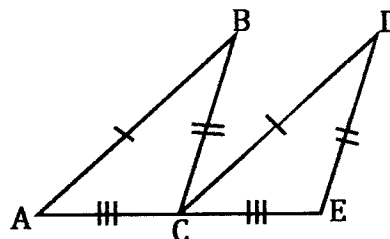
14. Given: $\overline{ST} \parallel \overline{VW}$, and $\overline{ST} \cong \overline{VW}$



Prove: $\overline{SU} \cong \overline{WU}$

Statements	Reasons
1.	1. Given
2.	2. Given
3.	3. Alternate Interior
4. $\angle SUT \cong \angle WUV$	4.
5.	5. AAS
6.	6.

15. Given: $\overline{AB} \cong \overline{CD}$, $\overline{BC} \cong \overline{DE}$, and $\overline{AC} \cong \overline{CE}$



Prove: $\angle A \cong \angle DCE$

Statements	Reasons
1. $\overline{AB} \cong \overline{CD}$	1.
2. $\overline{BC} \cong \overline{DE}$	2.
3.	3. Given
4.	4.
5.	5.